THE ARBITRAGE MODEL OF SECURITY RETURNS: AN EMPIRICAL EVALUATION

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Over the last two decades, the Capital Asset Pricing Model (CAPM) has emerged as the dominant theoretical basis for much of the research in financial economics. Because direct observation of the market portfolio is a pre-requisite for any valid application of the CAPM, it cannot serve as a theoretical basis for empirical research in securities markets. The Arbitrage Pricing Theory (APT) is a theoretical alternative to the CAPM in which the market portfolio plays no particular role. The purpose of this research is to develop and test a model of the security return generating process based on the APT.

Particular emphasis is placed on two facets of the proposed arbitrage model. First, the central prediction of the APT is an absence of arbitrage opportunities, the empirical identification of which would lead to a rejection of the theory. Thus, the first use to which the model is put is the examination of abnormal performance for the securities individually and jointly. The second application involves an event study comparison of the arbitrage model and a popular variant of the market model. The objective of this comparison is to establish the stability and usefulness of the arbitrage model against a known benchmark. In light of the growing list of empirical anomalies associated with the market model and the difficulties in application of the CAPM, an empirically tractable and theoretically sound model of security returns would be a significant step forward in financial research.

The data used in the study are daily returns for individual securities from the CRSP file and cover the period 1962 through 1979. The results indicate substantial support for the APT and the arbitrage model. Significant arbitrage opportunities are found to occur in less than 1% of the individual cases, and the hypothesis of jointly zero abnormal performance cannot be rejected in any case. In the event study comparison, the arbitrage model was found to work at least as well as the market model in all cases and was markedly superior in accounting for the January effect.

CHAPTER I
ESSENTIALS OF THE ARBITRAGE MODEL

Introduction

In the broadest sense, the primary concern of research in financial economics is the relationship between risk and return in well-organized markets. While security returns can generally be measured with relative ease, the determination of an appropriate measure of risk is a far more difficult question. Over the last two decades, the Capital Asset Pricing Model (CAPM) has emerged as the dominant theoretical basis for much of the research in this area. The fundamental result of the CAPM is straight-forward: the relevant riskiness for any asset is determined by the standardized covariance of its return with the return on the market portfolio, i.e., the portfolio consisting of all risky assets held in proportion to their value.

As a theory, the CAPM is extremely powerful and broadly applicable; however, no valid test of its empirical content has appeared in the literature. For reasons discussed in Roll (1977), such a test requires that the return on the market portfolio be observed directly. Because it is not technologically possible to obtain the necessary data, it is unlikely that a valid test will be forthcoming. For the same reason, any attempt to estimate the parameters of the model introduces bias of unknown magnitude and direction.

The impetus for this research stems from the need for a model free of these deficiencies. The purpose of this thesis is to develop and test an empirically tractable model of security returns which retains the intuitive appeal of CAPM-based models without the need for the market portfolio in estimation. In the next section, the theoretical basis for such a model is outlined.

An Alternative to the CAPM

The CAPM is a general equilibrium model of perfect markets with homogeneous investor expectations. In such markets, the CAPM will hold if investors have quadratic preferences or asset returns possess multivariate normal distributions. When these conditions are imposed, several important results follow. In particular:

1. An asset's expected return is independent of its own volatility; only that portion of its riskiness which cannot be diversified away is relevant.
2. All assets with the same non-diversifiable risk have the same expected return.
3. Asset returns contain two elements, one which is related to changes in the macro-economy and one which is unique to the particular asset. It is the unique portion which is eliminated by diversification.

These propositions collectively form the basis for much of the modern theory of finance. Curiously, these propositions are often used in informal derivations to justify the CAPM (see, for example, Brigham (1983), pp. 158-169). However, if the validity of these results is assumed *a priori*, the CAPM is needlessly restrictive. If securities markets are characterized by risk-averse investors who make decisions based only on expected returns and risk, then the assets will be priced as substitutes and the first two results are no more than simple economic propositions. Any asset which offered compensation for diversifiable risk would have its price bid up until the premium was eliminated. If two assets possessing the same non-diversifiable risk had different expected returns, then investors would sell (or supply) the one with the lower return and demand the one with the higher return. The relative prices would adjust until the expected returns were equal. Moreover, these conditions would hold across any subset of securities. Finally, that the unique portion of security returns can be eliminated by diversification is a property of any collection of imperfectly correlated variables. A certain portion will generally not be diversifiable simply because, to a greater or lesser extent, all asset returns depend on general economic conditions.

Ross (1976, 1977) has formalized the kind of reasoning outlined above in his Arbitrage Pricing Theory (APT). The principal assumption of the APT is that investors homogeneously view the random return, $\tilde{r}_i$, on the particular set of assets under consideration as being generated by a k-factor model of the following form:

$$\tilde{r}_i = E_i + \tilde{\delta}_o + b_{i1}\tilde{\delta}_1 + \ldots + b_{ik}\tilde{\delta}_k + \tilde{e}_i \quad i = 1, \ldots, n \qquad (1.1)$$

where

E_i = the expected return on the i^{th} asset

$\tilde{\delta}_o$ = the change in the pure interest rate. $E[\tilde{\delta}_o] = 0.$[1]

$\tilde{\delta}_j$ = The random value of the j^{th} common factor.

$E[\delta_j] = 0, j = 1, \ldots, k.$

b_{ij} = the sensitivity of the return on asset i to factor j.

$\tilde{e}_i$ = the random (unsystematic) portion of $\tilde{r}_i$. $E[e_i] = 0.$

also

$$E[e_i | \delta_j] = 0$$

$$\mathrm{COV}(\tilde{e}_i, \tilde{e}_j) = \begin{cases} 0 & i \neq j \\ \sigma^2_{ei} < \infty & i = j \end{cases}$$

Intuitively, the APT models security returns as a linear function of some unspecified state variables plus a random component. By appeal to the law of large numbers, any well diversified portfolio will have virtually no unsystematic risk. It is interesting to note that any linear model (including the CAPM) is a special case of the APT. In this sense, the APT is, as Brennan (1981) has remarked, " . . . a minimalist model since it predicts no more than the absence of arbitrage opportunities . . . [and] is logically prior to our other utility-based models" (p. 393).

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[1] Ross's formulation omits this term, implicitly assuming a constant risk-free rate. Including it allows for the absence of a risk-free asset, and is similar to Black's (1972) concept of a "zero-beta" portfolio. This issue is discussed in detail in Chapter III.

Testing the APT

Because the APT only predicts an absence of arbitrage opportunities, the identification of such opportunities would lead to a rejection of the theory. An arbitrage opportunity amounts to a constant non-zero portion of return not explained by the factors. In efficient markets, there are two fundamental no-arbitrage properties. First, portfolios with no net investment and no systematic risk must, on average, have no return. Second, portfolios with net positive investment and no systematic risk must have expected returns equal to the pure time value of foregone consumption. The return on such portfolios should equal the risk-free rate if such an asset exists; however, the existence of a risk-free asset is not a requirement of the APT.

A test of the APT requires the estimation of the parameters of eq. (1.1). Referring to eq. (1.1), if it is assumed that the random portion of return is completely eliminated, then the no-arbitrage propositions imply the existence of k + 1 weights such that

$$E_i = \lambda_o + \lambda_1 b_{i1} + \ldots + \lambda_k b_{ik}, \tag{1.2}$$

where λ_j is the risk premium on the j^{th} factor and λ_o is the expected return on all portfolios with no systematic risk (this result is formally demonstrated in Chapter III). While the APT provides no insight as to the interpretation of the factor risk premia, it is possible to re-write (1.2) in a more useful form. Consider a portfolio formed such that $b_{P1} = b_{P2} = \dots = b_{Pk} = 0$. If the portfolio has positive investment, its expected return is

$$E = \lambda_0 .$$

Next, a portfolio is formed with the property that its return is equal to the risk premium on the first factor; i.e., it is constructed such that $b_{P1} = 1$ and $b_{P2} = \dots = b_{Pk} = 0$. If it has no net investment, its expected return is

$$E^1 = \lambda_1 .$$

Repeating this process for every factor, equation (1.2) can be written

$$E_i = E^0 + E^1 b_{i1} + \dots + E^k b_{ik} . \qquad (1.3)$$

Substituting (3) into (1) and defining $\tilde{E}^i$ as $E^i + \tilde{\delta}_i$, then

$$\tilde{r}_i = \tilde{E} + \tilde{E}^1 b_{i1} + \dots + \tilde{E}^k b_{ik} + \tilde{e}_i . \qquad (1.4)$$

Equation (1.4) is an empirically useful representation of the APT: here the ex post return on the i^{th} security is expressed as a linear combination of the "zero beta" return ($\tilde{E}^0$) and the returns on the k arbitrage portfolios. Again, if k is taken to be one and $\tilde{E}^1$ is interpreted as the market risk premium, then (1.4) is the ex post two-parameter CAPM (Black 1972)).

Assuming that the returns on the k + 1 arbitrage portfolios can be determined (discussed in detail in Chapters III and IV), it is possible to test the APT. To accomplish this, the returns on n assets and the arbitrage portfolios are collected for some time period. Then for each security, a time-series regression is estimated of the form

$$\tilde{r}_i = \hat{\delta}_i + \hat{b}_{0i}\tilde{E}^0 + \hat{b}_{1i}\tilde{E}^1 + \dots + \hat{b}_{ki}\tilde{E}^k + \tilde{e}_i \qquad (1.5)$$

Because $\tilde{E}^0$ measures the "zero beta" return, $\hat{b}_{oi}$ should equal unity.

The intercept term, α_i, can be interpreted as a measure of abnormal performance and should not be significantly different from zero. A simple test of the predictions of the APT would consist of estimating the parameters of (1.5) subject to the constraints $\alpha_i = 0$ and $b_{0i} = 1$. The restricted estimate can then be compared to the unrestricted results using a standard F-test. If the constraint is binding in a substantial number of cases, then the APT may be rejected in that its predictions would be inconsistent with the data. Such a procedure, while intuitively appealing, suffers from at least two drawbacks. This approach has no objective decision rule. If the hypothesis were rejected in, say, 40% of the trials, would it then follow that the APT is invalid? Secondly, this approach requires that the contemporaneous residual covariances between returns be equal to zero. While this is formally an assumption of the theory, eq. (1.2) can be expected to hold as an approximation so long as the residuals are sufficiently independent for the law of large numbers to be operative. Hence, small, though significant correlations are not precluded. A number of large correlations would be indicative of an omitted factor(s).

For the reasons outlined above, a valid test of the APT requires that the cross-sectional dependence among the parameters be considered. Whether or not eq. (1.2) holds exactly is largely irrelevant (and probably untestable). With this theory, as with any theory, it is the extent to which its predictions are consistent with observed phenomena that is of interest. Brennan (1981) has remarked "[For an adequate test] . . . [w]hat is required is a test of the hypothesis that the intercept terms for all securities are equal to zero, though such a test may be difficult to construct" (p. 393). A consistent pattern of non-zero intercepts would be indicative of arbitrage opportunities, a result at odds with the arbitrage theory (and, for that matter, most of modern portfolio theory). Thus, a test of the APT amounts to testing whether the intercept terms are jointly different from zero, i.e., a pooled time-series and cross-sectional approach. Such a test is particularly appealing because, as shown in Chapter IV, it is formally equivalent to testing the following:

H_0: there exists no well-diversified portfolio with zero systematic risk and zero net investment which earns a significantly non-zero return.

vs.

H_A: such a portfolio exists.

This is a powerful test; if, for any collection of assets, a single arbitrage portfolio (out of an arbitrarily large number) can be identified with a non-zero return, the APT will be rejected. This is a strongly positivist test as well. The null hypothesis is literally the central prediction of the theory; thus, it is strictly the content of theory which is examined, not the assumptions. On the other hand, the theory is tested against an unspecified alternative; moreover, the test is conditional on the measurement of systematic risk. As a result, rejecting the theory does not necessarily invalidate the model. If the view is adopted that "it takes a model to beat a model," then the return generating function of eq. (1-4) is interesting in its own right. In the next section, the use of the model as an alternative to current practice is discussed.

The Arbitrage Model as a Tool in Financial Research

In financial research it is often desirable to specify a model of security returns which controls for the differential riskiness of the assets. Once this is accomplished, it is possible to analyze the effect of other variables (e.g., dividend yields) or events (e.g., unanticipated information) on security returns. To this end, the so-called "market model" has been widely employed (see the June, 1983 Journal of Financial

Economics for some recent examples). The return generating process specified by this model may be written

$$\tilde{r}_i = E_i + (\tilde{r}_m - E_m)b_i + \tilde{e}_i, \quad (1\text{-}6)$$

where $(\tilde{r}_m - E_m)$ is the deviation of some broad-based market index from its expectation.

The popularity of the market model can probably be traced to its simplicity, intuitive appeal, and similarity to the theoretical CAPM. However, as pointed out by Ross (1976) and more fully developed by Roll (1977, 1978), this similarity is more apparent than real. The model is in many ways closer to the APT than the CAPM; nonetheless, numerous shortcomings have been identified in the market model's ability to explain returns (e.g., Ball (1978), Banz (1981), Basu (1977), Reinganum (1981a)).

The arbitrage model of eq. (1-5) is an empirical alternative to the market model. Unlike the market model, the arbitrage model has a solid theoretical basis while retaining a certain simplicity and intuitive appeal. Thus, a comparison of the usefulness of the arbitrage model with that of the market model is a logical step.

One of the more popular uses of the market model has been the residual analysis methodology pioneered by Fama et al. (1969). Mandelker's (1974) study of the gains from mergers and Jaffee's (1974) research into the value of inside information are prime examples. The ability of this methodology to detect abnormal performance (systematic price changes unexplained by overall market movements) has been studied by Brown and Warner (1980). Their simulation results indicate that the procedure works quite well when the event date is known.

Studies of stock price behavior around various types of events are based on market efficiency. In an efficient market, prices should adjust rapidly and fully to new information. In this study, the residual behavior of the two models is compared around several known events. This comparison addresses two issues. First, because the market model is known to perform well in this type of study, the substantive results from the arbitrage model should be similar. Second, the consistency of the two models with the concept of efficient markets is of interest. The more consistent model would show greater pre-event adjustment, more rapid adjustment about the event date, and less drift subsequent to the event. This comparison also addresses the issue of stability of the estimated parameters. To the extent that the arbitrage model provides better resolution of the information in the residuals, it may judged to be a superior model of the return generating process.

Summary and Overview

The objective of this dissertation is twofold. First, Shanken (1982) has argued that no truly valid test of *any* theory of asset returns has appeared in the literature. The methodology employed in Chapter IV to test the APT is free of the problems identified in previous work and is actually quite general. Similar approaches could have broad applicability. Second, the market model suffers from both theoretical and empirical deficiencies. An alternative model with a stronger theoretical foundation and better empirical properties would be a significant step forward in financial research.

The present study is organized in six chapters. This chapter, the first, constitutes a brief outline of the need for research in this area and procedures by which it can be accomplished. Chapter II is a review of the relevant prior research in multi-factor models. Chapter III develops both the APT and the arbitrage model, as well as outlines the methodologies to be employed. In Chapter IV, the results of the tests of the APT are presented. In Chapter V, the empirical properties of the model as an alternative to the market model are evaluated and reported. Chapter VI summarizes the major findings, suggests topics for future research, and concludes this study.

CHAPTER II
PREVIOUS RESEARCH IN MULTI-FACTOR MODELS

Introduction

The Arbitrage Pricing Theory outlined in Chapter I provides a theoretical foundation for asset pricing without the stringent general equilibrium restrictions of the CAPM. Despite the theoretical justification and intuitive reasonableness of multi-factor models, empirical research has been dominated by the single-index "market" models. An extensive literature exists on the statistical properties of the model itself, and a number of authors have employed the model as a means of controlling for differential asset riskiness or general market conditions. Despite the popularity of this approach, research has been undertaken in three areas directly related to the arbitrage model. These areas are (1) purely empirical applications of multivariate statistical techniques (principally cluster and factor analysis), (2) multivariate regression models based on *a priori* assumptions as to the number and identity of the relevant factors, and (3) tests of the APT. Much of this research preceded the development of the APT and it is interesting to re-examine the empirical results obtained in an arbitrage model context. The next three sections examine this research and its implications for the arbitrage model.

Applications of Multivariate Statistical Techniques

When a group of variables exhibits a high degree a linear correlation or "redundancy," several dimension-reducing techniques are available to summarize the data in a more parsimonious fashion.[1] Because security returns are often highly correlated, cluster and factor analysis have been applied in efforts to establish the existence of an underlying structure in the data. With either technique, it is hypothesized that the variables are elements of a k-dimensional subspace, where k is "small" relative to the numbers of variables. In either case, k is unknown *a priori*.

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[1] What follows is intended as a very brief, intuitive description. A good introduction to cluster analysis may be found in Elton and Gruber (1970). Factor analysis is taken up in detail in the next chapter.

With cluster analysis, the objective is to assign each variable to one of k homogeneous groups. In its simplest form, a cluster analysis of security returns begins with a full rank correlation matrix of returns. The two securities with the highest correlation are combined into a single variable, thereby reducing the rank of the correlation matrix by one. The correlation matrix is then recomputed with the new variable and the reamining n-2 securities. The two variables with the highest correlation in the new matrix are combined and so on. The process is continued in an iterative fashion until no significant correlations remain between some number of "clusters." However, no completely objective rule exists for determining the appropriate number of clusters.

In the general factor analysis model, security returns are assumed to be characterized by a set of hypothetical or latent variables. The returns are expressed as a linear combination of these variables plus a random (or unique) portion.[2] Like cluster analysis, factor analysis usually begins with the estimated correlation matrix. Using one of several techniques, an estimate of the percentage of total variance which is unique is obtained for each asset. The main diagonal of the correlation matrix (consisting of ones) is adjusted by subtracting this estimated "uniqueness." The result for a particular asset is an estimate of its "communality," i.e., that portion of its total return which is systematic. If the errors are assumed to be uncorrelated across securities, then the resulting adjusted correlation matrix can be interpreted as an estimate of the common intercorrelation. The next step is to construct an artificial variable which accounts for a maximum of the common variance. Next, a second variable (generally constrained to be orthogonal to the first) is constructed which accounts for a maximum of the remaining variance. This procedure is continued, yielding k variables which account for all the estimated common variance.[3] Several objective criteria are available for determining k. A discussion of these is deferred to Chapter III.

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[2] Principal component analysis differs from factor analysis. In component analysis, no distinction is made between random and non-random portions. This point is discussed in the context of research which has used this approach.

Both cluster and factor analysis are generally employed as explanatory techniques and results obtained thereby are purely empirical. However, if the elements of a particular cluster have similar characteristics, it may be possible to formulate hypotheses for further testing. Similarly, if a given factor is particularly related to some group of securities, it may be possible to infer the identity of the factor. Regardless of the validity of such heuristics, the research examined below relates to the existence of multiple factors in security returns as well as the number of relevant dimensions.

One of the earliest studies to employ dimension reducing techniques was that of Farrar (1962). Farrar applied the principal component approach to 47 industry groups in an effort to create a relatively small number of asset groups with low first-order correlations. He found that the first five components accounted for about 97% of the total joint variation among the industry groups, with the first component capturing 77% of the total. Examination of his results (p. 41) indicates the presence of a single, dominant factor with at least two additional significant factors.

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[3] If the factor model fit perfectly, the reduced correlation matrix would be rank k. In practice, k is regarded as the approximate rank, allowing for measurement error and non-linearities.

The principal component approach was also applied by Feeney and Hester (1967). The purpose of their research was to objectively develop weights for a stock market index. Using the 30 securities in the Dow Jones Index, they found that the first two components (of the covariance matrix) accounted for 90% of the total variance, with the first component accounting for 76%. Interestingly, the correlation between the Dow Jones Index and the first component was found to be in excess of .99. The results from the component analysis are nearly identical to those found by Farrar, despite the different samples and time periods employed.

In 1966, King investigated the nature of the latent structure of security returns. His work is of particular importance because he recognizes both the presence of a market factor and the existence of unsystematic (or unique) effects. The explicit purpose of King's study was to determine whether inter-relationships among security returns could be attributed to a market factor and an industry factor corresponding to a two-digit SIC classification. Using a sample of 64 stocks in six industry groups, King performed both a mixed factor/cluster analysis and a multi-factor analysis. In the mixed analysis, he extracted the first factor (the market factor) and clustered the remaining variation. When the maximum correlation between groups dropped below .20, the group corresponded exactly to the SIC two-digit classifications. When a seven factor solution was obtained, the same pattern emerged; all securities were sensitive to the general market effect and an industry factor. Also, King found that the first factor accounted for 74% of the estimated total systematic variation; however, his results differ from those of Farrar and Feeney and Hester in that the subsequent factors (particularly the second and third) were not as pronounced. Also, the relative importance of the market factor in explaining the systematic or common variation was found to decline over time, from a high of 63% in the sub-period June 1927 to September 1935 to a low of 37% for the period August 1952 to September 1960.

At the time of King's study (1966), the Sharpe (1963) single-index model was gaining popularity as a simplification to the general Markowitz (1959) portfolio problem. The validity of this model hinges on the absence of contemporaneous residual correlations among the assets. King's findings are at odds with this requirement. In a 1973 study, Meyers extended King's methodology to include less homogeneous industry groups, as well as the time period 1961-1967. After extracting the first principal component, Meyers clustered the residual correlation matrix and found results generally supportive of King's; however, he does identify a weakening of the industry effects. Meyers then extracted six components from the residual correlation matrix and reported evidence of industry effects similar to King's, though with significantly less clarity. Meyers concludes that King's results overstate the importance of industry effects, but he concurs in the finding of residual covariance unexplained by a general market effect.

The relative strength of industry effects was examined in 1977 by Livingston. In this study, a number of important issues are identified;

in particular, Livingston documents that the principal components approach is inappropriate in that it tends to extract more common variance than actually exists.

To determine the magnitude of industry effects, Livingston proceeded to regress returns from 734 securities (in over 100 industries) on the S & P Composite Index return. Next, the residual correlation matrix was examined for significantly non-zero correlations. Within industries, 20% of the correlations were found to be significantly different from zero, with very few negative elements. Across industries, 6% were significantly positive and 2% significantly negative. However, some of the industries examined showed little residual correlation. Livingston concludes that a single-index model ignores a significant portion of the co-movement in security returns and that the use of industry indices should improve the results. Such models have been constructed and are reviewed below.

The most general conclusion which can be drawn from the King, Meyers, and Livingston research is that extra-market covariation does exist, but it is not clear whether the effect is related to industry classification *per se*. An alternative explanation could be offered to the effect that certain types of businesses are particularly sensitive to different macro-economic factors. If this proposition is correct for the members of a homogeneous industry group, then an "industry effect" will appear to exist. Because factors such as interest rates, foreign exchange rates, inflation, input prices (raw materials and wages), and so on do not move in lockstep, firms with particular dependencies on any one factor will exhibit "extra-market" influences. This is simply due to the averaging implicit in the construction of a market index.[4]

Studies by Farrell (1974, 1975) and Arnott (1980) have used cluster analysis to define groups of securities in terms of their return characteristics as opposed to industry classification. Farrell used a stratified (across industries) sample of 100 securities. He computed the residual correlation matrix from a single-index model. These residuals were clustered until no correlation above .15 remained. The results of this procedure were four clusters which Farrell labels as growth, stable, cyclical, and oil. Arnott used 600 securities and a somewhat less stringent rule to halt the clustering process. His results indicate five clusters which he labels quality growth, utilities, oil and related, basic industries, and consumer cyclicals. The results of the two studies are actually quite similar; the primary difference is that the Farrell study combines the utility, basic industry, and consumer cyclical into two clusters, the stable and cyclical. Both of these studies are generally supportive of a multi-factor model, where the factors are some set of macro-economic variables rather than simple industry effects.

The multivariate studies reviewed in this section have, in varying degrees, a similar result: a single index model ignores potentially useful information about the co-movement of security returns. The techniques used in these analyses are all forms of correlation analysis; no model or theory is employed. In the next section, several models which attempt to incorporate extra-market information are examined.

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[4] In the case of a value-weighted index, the averaging is in terms of the characteristics of the largest firms versus the most numerous in the case of an equal-weighted index.

Multiple Regression Models of Security Returns

Several authors have sought to improve the single-index model by including additional variables. In an early effort, Kalman and Pogue (1967) compared the ability of single and multiple index models to recreate the Markowitz efficient frontier and to predict correlation matrices. Their results indicate little, if any, benefit from a multi-index approach. Farrell (1974) criticizes the method used by Kalman and Pogue in constructing the multiple indices, attributing the lack of success to the high degree of collinearity among the industry indices. Using the relatively uncorrelated clusters (described in the previous section) in addition to a general market effect, he reports superior results when compared to a single index formulation.[5]

In another study examining the ability of various models to predict correlation matrices, Elton and Gruber (1973) test ten different models of security returns. They find that three models outperform all other techniques, including the single index and several multiple index models. The three models differ in their assumptions concerning the pattern of correlation coefficients. The overall mean model sets all coefficients equal to the average. The traditional industry mean sets all correlations within an industry equal to the industry average, and all inter-industry correlations are set equal to their average. The third model is the same as the traditional industry with the exception that industries are defined by a principal component solution ("pseudo-industries"). Elton and Gruber's results indicate that superior forecasting is possible using information not produced by index models. Unfortunately, their multi-index models are based on principal component solutions and the assumption of zero residual correlations is inappropriate.

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[5] Farrell extracts the market effect by regressing the cluster returns on a market index and using the residuals as "explanatory" variables. This procedure creates orthogonal indices by construction; however, such an approach is suspect on econometric grounds. It is difficult to justify the use of random noise (i.e., the residuals) from one estimation as "explanatory" variables in another.

Other studies have used information beyond a general market effect in estimation. Rosenberg (1974) assumed the general validity of the single index approach, but he used a number of firm-specific "descriptor" variables to obtain forecasts of the parameters. Lloyd and Schick (1977) have tested a two index model proposed by Stone (1974), where the additional index is composed of debt instruments. Langetieg (1978) adopted an approach similar to Farrell's, using orthogonalized industry indices to measure gains from mergers. All of these studies find benefits in the use of extra-market information, but they lack a theoretical underpinning. The arbitrage theory provides this missing element, and studies incorporating it directly are reviewed in the next section.

Tests of the Arbitrage Theory

The first published study of the APT is credited to Gehr (1975). Gehr constructed two samples of 360 monthly returns, one consisting of 24 industry indices and the other of 41 individual companies. He next obtained a three component solution for the 41 companies. The industry returns were then regressed on the components to estimate the sensitivity coefficients. A second-pass regression of the mean industry index returns against the coefficients was performed as the final step. Of the estimated risk premia, only one is found to be generally significant.

An empirical anomally associated with the market model has been investigated by Reinganum (1981b) and Banz (1981). When portfolios are formed based on firm size, small firms earn significantly greater rates of return, even after accounting for difference in estimated betas. Reinganum (1981a) has examined the same question using an arbitrage model. Essentially, Reinganum forms a set of control portfolios based on ranked factor loadings. Then, the returns on the control portfolios are subtracted from corresponding individual security returns. The resulting excess returns are ordered into deciles based on market equity values, and the average excess return is computed for each decile. Reinganum's results are similar to those found using the market model: portfolios of small firms offer a risk-adjusted return significantly greater than the portfolios of large firms. Thus, Reinganum rejects the arbitrage model as an empirical alternative to the simpler market model.

Oldfield and Rogalski (1981) have examined the influence of factors estimated from Treasury bill returns on common stock returns. As a first step, they gather Treasury bill returns for 1 to 26 week maturities. The one week return is then subtracted from the subsequent maturities to calculate excess weekly returns. The one week rate is reserved for the risk-free rate. Next, the excess T-bill returns are factored and factor scores are computed.[6] Next, individual common stock returns are regressed on the factor scores, yielding a set of sensitivity coefficients. The stocks are then randomly assigned to intermediate portfolios, and the covariance matrix of the returns among the portfolios is calculated. Using this covariance matrix, a minimum variance portfolio is calculated for each factor with the property that a particular portfolio is sensitive to that factor, with a zero loading on the others. Additionally, a minimum variance portfolio is formed with no sensitivity to any factor

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[6] Factor scores are estimates of the population factors; hence they constitute a time-series of measurements of the factors.

(a "zero-beta" portfolio). The weekly returns on these factor portfolios is computed, and these are used in time-series regressions to re-estimate sensitivity coefficients.

Their first result from the procedure is that significant correlation exists between common stock returns and the factor portfolio returns. Next, the authors run a cross-sectional regression of the weekly intermediate portfolio returns and their factor loadings in each of 639 weeks. They then compare the mean regression coefficient for a particular factor with the mean return on the factor portfolio. They argue that the two should be equal, and find no statistical difference. By including an equal weighted market portfolio, the authors find that the significance of the factor portfolios is greatly diminished, a result which they attribute partially to the collinearity between the variables. The authors report that the estimated risk-free rate is significantly less than the corresponding T-bill rate, while the cross-sectional intercepts are not different from zero.

Fogler, John, and Tipton (1981) have also attempted to relate the returns on debt and equity instruments in the context of the arbitrage theory. The basic data for this study were excess monthly returns on 100 securities divided into seven groups. The first four groups were selected on the basis of Farrell's cluster analysis, consisting of stocks classified as growth, stable, cyclical, and oil. The other three groups correspond to the pseudo-industries developed by Elton and Gruber (both studies are reviewed in a previous section). The authors next calculate excess monthly returns on a value-weighted market index, a three month Treasury bond index, and a long-term Aa utility bond index. The excess returns were calculated by subtracting the return on a one month Treasury bond. Next, the excess returns on the securities were regressed on the three indices; of the three, only the market index had generally significant coefficients. The authors report that some of the groups display consistent signs on other indices; however, no non-parametric results were included.

In a second part of their study, Fogler, John and Tipton extract a principal component solution from the 100 securities, retaining the first three. They then examined the canonical correlation between the components and the three indices. From this analysis, one important result emerges: the correlation between the three components and the market index is nearly perfect. Also, in some sub-periods there is a statistically significant relationship between the components and the three month Treasury bond yield. Whether or not the authors have achieved their goal of "imparting economic meaning to the stock returns factors" (p. 327) is difficult to say; yet they implicitly establish an important empirical result; namely, the return on the overall market can be decomposed without loss of information about the market while potentially including other relevant information. Thus, while their study is not actually a test of the APT, it nonetheless suggests a certain empirical rationale for the theory.

A final study deserving of particular attention is that of Roll and Ross (1980). This study is a straightforward extension of Gehr's methodology. The authors form 42 portfolios of 30 securities each, using ten years of daily security returns. A factor solution is then obtained for each group. For each group, a cross-sectional GLS regression of mean returns on the factor loadings is estimated. The authors report that at least three factors of the five used are "priced" in the results. Next, an additional variable, the standard deviation of return, is included in the cross-sectional regressions. After correcting for the positive dependence between sample mean and sample standard deviation arising from the positive skewness in daily returns (by using non-overlapping samples), little support is found for the hypothesis that returns are related to total volatility. As a final test, Roll and Ross test for cross-sectional differences in the intercepts from the cross-sectional regressions. To do so, they employed Hotelling's T^2 statistic to account for cross-sectional dependencies in the estimates. Their results indicate no significant difference, lending support to the APT.

Summary

The proceding three sections have reviewed research in three areas--purely empirical analysis of stock market groups, multiple regression models based on *a priori* knowledge of the relevant variables, and studies testing the APT, either directly or indirectly. Of the multivariate studies, the results obtained from a variety of different approaches are consistent in that they generally indicate that a single index model ignores significant facets of security returns. This conclusion is reinforced by the multiple regression studies in that the additional variables specified add significant explanatory or predictive power despite their *ad hoc* nature.

The APT offers, in principle, an empirical alternative. The studies published to date using it all suffer from serious methodological flaws; in addition, no tractable multi-index model based on the APT has been forthcoming. Because many of the methodological problems in the literature stem from a misapplication of factor analysis, a discussion of them is deferred to the next two chapters where the application of factor analysis to security returns is addressed. Problems also arise in the development of testable hypotheses in an arbitrage pricing framework and with the nature of the appropriate return generating function. These three issues--factor analysis of security returns, testable hypotheses of the APT, and the structuring of a return generating function--are inter-related to the extent that the validity of any one of the three depends on the other two. In other words, the theoretical justification for a multi-factor return generating function obtained from a factor analysis is found in the APT. However, a test of the APT requires a return function obtained from a factor analysis procedure. Finally, a number of factor analysis procedures are available; the choice of a particular one depends on both the APT and the desired form of the empirical model derived therefrom. The next chapter considers each of the subjects independently before combining them into the arbitrage model.

CHAPTER III
THE ARBITRAGE MODEL: THEORY AND ESTIMATION

Introduction

In the previous two chapters, the need for an alternative model of security returns was established and evidence for the validity of a multi-factor representation was examined. In the first section of this chapter, the theoretical basis for such a model is illustrated. In the second section, the relationship between the APT and the general factor analysis model is developed. The results of these sections are used to derive an empirical model of returns and to establish the testable hypotheses of the APT.

The Arbitrage Pricing Theory

The APT was originally proposed by Ross (1976, 1977). A simplified approach was derived by Huberman (1982). The theory has been generalized and extended by Ingersoll (1982). The exposition in this section draws heavily from these three sources.

The principal assumption of the APT is that investors homogeneously view the random returns, $\tilde{r}$, on the particular set of assets under consideration as being generated by a k-factor linear model of the following form:[1]

$$\tilde{r} = E + B\tilde{\delta} + \tilde{e}, \tag{3-1}$$

where

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[1] Strictly speaking, complete homogeneity of investor expectations is not required. Ross (1976) has established that the existence of non-negligible agents with upward bounded relative risk aversion and homogeneous opinions about expected returns are sufficient. As Ross notes, however, translating *ex post* occurences into *ex ante* anticipations will require homogeneity.

$E_{n \times 1} = \langle E_i \rangle$ = the expected returns on the n assets

$B_{n \times k} = \langle b_{ij} \rangle$ = the sensitivity of the return on asset i to changes in common factor j (factor loadings)

$\tilde{\delta}_{k \times 1} = \langle \tilde{\delta}_j \rangle$ = the random values of the k common factors

$\tilde{e}_{n \times 1} = \langle \tilde{e}_i \rangle$ = the random (unsystematic) portion of r.

It is also assumed that

$E[e] = \underline{0}$

$E[\delta] = \underline{0}$

$E[e\delta'] = \underline{0}$

$$E[ee'] = \psi = \begin{cases} \bar{\sigma}^2_{ij} < \bar{\sigma}^2 & i = j \\ 0 & i \neq j. \end{cases}$$

In other words, the deviation of the return on asset i from its expectation is a linear combination of the random values of the k factors and a unique, residual component. The residuals are assumed to be independent of the factors and mutually uncorrelated. In the absense of a riskless asset with a constant certain return, eq. (3-1) may be written

$$\tilde{r} = E + A\tilde{\delta} + \tilde{e}, \tag{3-2}$$

where

$\tilde{\delta}_{(k+1) \times 1} = \langle \tilde{\delta}_j \rangle$ = the random values of the k common factors with $\tilde{\delta}_o$ as the change in the "zero beta" return

$A_{n \times (k+1)} = \langle \ell : B \rangle$ = The augmented factor loading matrix

$\ell_{n \times 1} = \langle 1 \rangle$ = the sum vector (a column vector of ones).

Heuristically, the arguments underlying the APT begin with the consideration of a portfolio vector, x, chosen such that $x'\ell = o$. The components of x are the dollar amounts invested in each asset. Since the total investment is zero by construction, all purchases (long positions) are financed by sales (short positions).[2] If x is a well-diversified portfolio with each x_i of order $1/_n$ in absolute magnitude, then by the law of large numbers, the dollar return on x is

$$\begin{aligned} x'\tilde{r} &= x'E + x'A\tilde{\delta} + x'\tilde{e} \\ &\simeq x'E + (x'A)\tilde{\delta}. \end{aligned} \tag{3-3}$$

If x is chosen to have no systematic risk as well, then the return is

$$x'\tilde{r} \simeq x'E. \tag{3-4}$$

Taking α to be any non-zero scalar, then αx is an arbitrage portfolio. If it is assumed that the random portion of (3-3) can be completely eliminated by diversification, then (3-4) holds with equality and it must be the case that

$$x'\tilde{r} = x'E = 0, \tag{3-5}$$

or unbounded certain profits are possible by increasing the scale (α) of the arbitrage operation. If this condition holds for all portfolios constructed in the manner described above, then there exist constants $<\lambda_o, \ldots, \lambda_k> = \lambda'$ such that

$$E = A\lambda, \tag{3-6}$$

where A is the augmented factor loading matrix. Algebraically, (3-6) is simply the statement that all vectors orthogonal (perpendicular) to A are orthogonal to E if and only if E is in the span of the columns of A. This result and several others can be illustrated by introducing the following notation:

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[2] In the absense of restrictions on short selling, such portfolios can always be constructed. Even with short selling restrictions, investors with positive net holdings can, in effect, engage in such activities by buying and selling. Letting w be the dollar amounts invested in the n assets (with $w'\ell = W$, the investor's net wealth), then, assuming no transactions costs, the difference between w and any other portfolio, $\hat{w}$, is an arbitrage portfolio: $w + x = \hat{w}$. Thus, an investor who changes his relative investments is implicitly purchasing an arbitrage portfolio.

$S = \text{span}\,\{A\}$, where A is assumed to have full column rank

$S^{\perp}$ = set of all vectors orthogonal to S with orthogonal basis

$$x = \langle x_1, \ldots, x_{n-k-1} \rangle.$$

By construction, then

$$S \oplus S^{\perp} = \mathbb{R}^n$$

$$S \cap S^{\perp} = \{\underline{0}\}$$

$$x_i' A = \underline{0} \qquad i = 1, \ldots, n-k-1$$

$$x_i' x_j \begin{array}{l} \neq 0 \\ = 0 \end{array} \qquad \begin{array}{l} i = j \\ i \neq j. \end{array}$$

Equation (3-6) follows from the no-arbitrage assumption; either $E \varepsilon S$ or arbitrage is possible. To see this, note that E can always be written

$$E = A\lambda + z, \tag{3-7}$$

where $z \varepsilon S^{\perp}$. But z is itself an arbitrage portfolio with return

$$z'E = (z'A)\lambda + z'z = z'z \neq 0. \tag{3-8}$$

So (3-6) must hold to prevent arbitrage.

Following Huberman (1982), the results obtained above can be extended to the case where the residual portion of return is not completely eliminated. The objective is to establish an upper bound on the sum of the squared deviations from the pricing relatinship (3-6). The APT considers a sequence of economies with increasing numbers of risky assets. The n^{th} economy has n risky assets whose returns are generated by a k-factor model, where k is a fixed number. Arbitrage is defined as the existence of a subsequence, z_n, of arbitrage portfolios with the properties

$$\lim_{n\to\infty} \quad z_n'E = {}^{+}\infty \tag{3-9}$$

$$\lim_{n\to\infty} \quad \text{var}(z_n'E) = 0. \tag{3-10}$$

Intuitively, arbitrage possibilities exist whenever increasing profits at diminishing risk are obtainable as the number of assets grows. Put another way, the reward to volatility ratio increases without limit. To preclude such occurrences, there must be an upper bound to

$$(E_n - A_n\lambda_n)' \, (E_n - A_n\lambda_n), \tag{3-11}$$

the sum of squared deviations in the n^{th} economy. Referring to (3-4) and assuming that z is scaled such that z'z increases to infinity with n, (the subscripts are understood)

$$z'\tilde{r} = z'E + z'\tilde{e}, \tag{3-12}$$

substituting (3-7) for z'E

$$z'\tilde{r} = z'z + z'\tilde{e}. \tag{3-13}$$

Letting α be a scalar between 1/2 and 1 and defining $\gamma = (z'z)^{-\alpha}$, then γz is an arbitrage portfolio with expected return and variance

$$\gamma z'E = \gamma z'z = (z'z)^{1-\alpha} = E[z] \tag{3-14}$$

and

$$\gamma z'E[ee']z\gamma = \sigma_z^2 \leq \bar{\sigma}^2 \gamma^2 (z'z) \leq \bar{\sigma}^2 (z'z)^{1-2\alpha}. \tag{3-15}$$

By construction

$$\lim_{n\to\infty} E[z] = \infty \tag{3-16}$$

$$\lim_{n\to\infty} \sigma_z^2 = 0. \tag{3-17}$$

For example, if α is taken to be 3/4, the expected return increases with the fourth root of z'z and the variance decreases with the square root. The reward to volatility ratio is

$$\frac{E[z]}{\sigma_z} \geq \frac{(z'z)^{1-\alpha}}{\bar{\sigma}(z'z)^{1/2-\alpha}} = ||z||\sigma \tag{3-18}$$

which does not have a lower bound unless z'z is bounded. From (3-7), $z = E - A\lambda$; hence the foregoing suffices to show that unlimited deviations from the pricing relationship (3-6) give rise to arbitrarily large profits. If such profits are precluded by assumption then it follows that

$$(E - A\lambda)' \, (E - A\lambda) \leq M. \tag{3-19}$$

Inequality (3-19) indicates that the permissible sum of squared deviations is less than some finite number for any number of assets. As a consequence, as the number of assets increases, the approximation improves. The reverse is, of course, for any finite set of assets, the approximation can be quite poor.

The existence of a finite bound on the arbitrage pricing relationship suggests a natural test of the APT. For A and E given, it is possible to determine λ such that the sum of the squared deviations from the pricing relationship is minimized, i.e., minimize

$$(E - A\lambda)'(E - A\lambda). \tag{3-20}$$

Equation (3-20) is simply the OLS estimate of E on B. The resulting sum of squared errors could be compared to the bound in (3-19) and the APT rejected if the bound is exceeded. Such a test, however, requires that an _a priori_ bound be established.

In the absence of such a specification, several authors (notably Roll and Ross (1980)) have attempted to verify the APT by establishing the linearity of (3-6). As pointed out by Shanken (1982), the linearity of (3-6) is not literally an implication of the APT for any finite collection of assets. For any such collection, (3-6) is an approximation and will have a finite bound on the sum of the squared errors. To take a polar case, in the absence of _any_ linear relationship for a particular set of assets, the bound would be equal to the sum of the squared expected returns. Such a result would not necessarily invalidate the APT because the permissible upper bound on the pricing relationship is not known _a priori_; hence, an examination of the degree of linearity in the pricing relationship is without power to reject the APT.

The central prediction of the APT is the absence of arbitrage opportunities where arbitrage is defined as a non-zero return on a well diversified portfolio with no net investment and no factor risk. The empirical identification of such opportunities would lead to a rejection of the theory. In the next section, the estimation of a k-factor model to be used in testing the APT is discussed.

Estimating the Arbitrage Model

The APT is a theory of the structure of asset returns. In the theoretical development of the previous section, it was assumed that the matrix of factor loadings (and the number of factors) was known. Because the APT provides no insight into the nature or number of the factors, it will be necessary to infer both from observed security returns. Techniques for accomplishing this fall under the general heading of factor analysis. In this section, a particular type of factor analysis due to Lawley (1940) is outlined, following Jöreskog (1967) and Lawley and Maxwell (1971).

It is easiest to conceive of factor analysis as a form of linear regression in which the number and identity of the regressors is unknown. Factor analysis is then a technique by which (for k regressors) the coefficients and variables of the linear model are simultaneously determined. Based on the assumptions of the previous section, the covariance matrix of returns is

$$\begin{aligned} E[rr'] &= E[B\delta + e)(B\delta + e)'] \\ &= BE(\delta\delta')B' + E[ee'] \\ &= B \, \phi \, B' + \psi, \qquad (3\text{-}21) \end{aligned}$$

where ϕ is the covariance matrix of the factors. Let Q be a matrix satisfying $Q'Q = I$ and $Q \, \phi \, Q' = I$.[3] Equation (3-21) can be written

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[3] Such a matrix exists for any symmetric matrix of full rank. The columns of Q are the eigenvectors of ϕ scaled by the square root of their respective eigenvalues. See Frieberg _et al_. (1979).

$$\begin{aligned} E[rr'] &= B(Q'Q)\,\phi\,(Q'Q)'B' + \psi \\ &= (BQ')(Q'B)' + \psi \\ &= B^*B^{*\prime} + \psi. \end{aligned} \qquad (3\text{-}22)$$

Thus, without loss of generality, it can be assumed that the factors are mutually uncorrelated with unit variances. When the factors have this relationship, the insertion of any full rank orthonormal matrix leads to a mathematically equivalent solution. Thus there is an infinity of mathematically equivalent factor loading matrices. A linear transformation of this type is termed a rotation and geometrically amounts to a rigid motion of the factor axes to a new set of coordinates. This indeterminateness is not a problem with the APT since no interpretation of the factors is necessary; researchers are free to choose any convenient orientation. Naturally, this lack of uniqueness would make the task of "identifying" the factors difficult and any interpretations of them suspect.

If it is assumed that the vectors δ and e follow multivariate normal distributions, then the elements of the sample covariance matrix S have a Wishart distribution with t - 1 degrees of freedom (t is the number of observations). In this case it is possible to obtain maximum likelihood estimates for B and ψ (again, for a given k). Following the usual technique of maximizing the log-likelihood function, it can be shown (Lawley and Maxwell (1971), p. 26) that an equivalent procedure is to minimize the function

$$F(B,\psi) = \ln|\Sigma| + tr(S\Sigma^{-1}) - \ln s \;\; - n, \qquad (3\text{-}23)$$

where Σ is the hypothetical covariance matrix and n is the number of variables. No direct solution exists, so numerical techniques are used to find the minimum value and the resulting estimates $\hat{B}$ and $\hat{\psi}$.

The principal attraction of the maximum liklihood approach is that it allows a test for the number of common factors. Denote by $L(\Omega)$ the maximum of the likelihood function for k unrestricted and let $L(\Omega_0)$ be the maximum under the null hypothesis of exactly k factors. If λ is the ratio of the restricted maximum to the unrestricted, it is well known (e.g., Mendenhall and Schaeffer (1973)) that $-2\ln\lambda$ converges in distribution to χ^2 with degrees of freedom equal to the number of parameters or functions of parameters assigned specific values under the null hypothesis. In the case of a factor analysis, the number of parameters estimated in the unrestricted model is the sum of n variances and the $1/2(n^2 - n)$ unique covariances, for a total of $1/2n(n + 1)$. There are nk unknowns in B and n unknowns in ψ. Without further restrictions, the matrix B is not uniquely defined. For computational reasons, it is convenient to require that $B'\psi B$ be a diagonal matrix. This has the effect of imposing $1/2k(k-1)$ restrictions; hence the total number of free parameters is $nk + n - 1/2k(k-1)$ and the degrees of freedom are

$$1/2n(n + 1) - nk - n + 1/2k(k - 1) =$$
$$1/2[(n-k)^2 - (n + k)]. \qquad (3\text{-}24)$$

In fact, the value of the computed χ^2 statistic is simply t times the minimand of eq. (3.23), explaining its use in the estimation procedure.

The maximum likelihood procedure provides for a test of $k = k_0$, where k_0 is a prespecified number of factors. In essence, the hypothetical covariance matrix constructed using only k_0 factors is compared to the saturated (sample) covariance matrix and if the discrepancy is found to be sufficiently small, the hypothesis is not rejected. In the usual exploratory case, it is not possible to specify a pre-determined value of k; instead, what is desired is an estimate of the dimension of the model. The procedure adopted is to begin with a small hypothesized value of k. If the hypothesis is rejected, k is increased by one and the test repeated. The dimension of the model is taken to be the smallest value of k which yields a non-significant χ^2 at a predetermined significance level.

Because the ultimately determined value of k depends on a sequence of prior tests, the assumptions of the classical Neyman-Pearson theory are violated. Thus, the test can only be interpreted as a test of sufficiency, and practice has shown that the value of k arrived at by this procedure using conventional significance levels is greater than the number of relevant factors; hence, the test should be regarded as conservative in the sense that it is unlikely to lead to an underestimate of the true number of factors (Lawley and Maxwell (1971), Harman (1976), Horn and Engstrom (1979)). With this approach, it is important to note that significant χ^2 values lead to the fitting of more factors; hence, greater significance levels will lead to fewer factors being retained. Horn and Engstrom's (1979) results from a related criterion indicate that, for large samples, significance levels in excess of .999 are warranted.

Because of the tendency for the maximum likelihood approach to over-estimate the number of substantially important dimensions, two other criteria will be examined. Akaike (1973, 1974) has proposed an information theoretic loss function as an extension to the likelihood approach and Schwarz (1978) has developed a large-sample Bayesian criterion. All three criteria are related. Schwarz (p. 461) indicates that his and Akaike's approach amount to adjusting the maximum likelihood estimator. If $M(k_j)$ is the value of the likelihood function for k factors, then Akaike's procedure results in the selection of k such that $\ln M(k_j) - \ln k_j$ is largest and Schwarz's criterion results in the selection of k such that $\ln M(k_j) - 1/2 k_j \ln t$ is maximized. In large samples, the three criteria can lead to very different estimates of k. Schwarz's criterion will lead to smallest estimate and the maximum likelihood procedure the largest.

The results from applying each of these standards to daily security returns are reported in the next chapter.

Several other points about the maximum likelihood approach are of interest. A consequence of the weighting scheme implicit in the procedure is that the resulting estimates are scale-free; hence, correlations may be used instead of covariances. In regard to the distributional assumptions underlying the approach, Howe (1955) has demonstrated that the same loadings and residual variances result from maximizing the determinant of the estimated common correlation matrix; thus the approach is valid as a descriptive measure regardless of the underlying distribution. The principal drawback relates to the computational resources required to obtain the solution. The CPU time required varies exponentially and appears to be proportional to the fourth or fifth power of the number of variables. Sample sizes are thus limited. Also, convergence of the numerical algorithms employed need only be local; thus the solution obtained may not be the global maximum. In any event, once the number of factors and their associated loadings are determined, it is still necessary to measure the factor risk premia. This topic is the subject of the next section.

Measuring the Risk Premia

The final step in constructing a multi-factor model of security returns is to use the information from the factor analysis to estimate the time-series behavior of the factor risk premia. In this section, the estimates of the factor loading matrix, $\hat{B}$, and the diagonal residual covariance matrix, $\hat{\psi}$, are taken to be fixed; hence the carats are dropped. Also, it is assumed that the estimates of B and ψ are obtained from the covariance matrix, so the typical element of B is the covariance of the return on security i with common factor j. The typical element of ψ is the uniqueness for asset i multiplied by its variance, i.e., the portion of its variance not associated with the common factors.

The approach adopted here is similar to methods used by Oldfield and Rogaliski (1981), though with some important differences. The general technique employed is to partition the observations into two groups: an estimation (base) period and a test period. The data from the base period are used to obtain a factor solution. Next, for each factor, a portfolio is constructed with unit sensitivity to that factor subject to the constraints that it use no wealth and that it have no correlation with the other factors. Also, a zero beta portfolio is formed possessing positive investment and no correlation with the factors. Without further constraints, the weight vectors for each of the portfolios are not unique; for any particular one there would n-k-1 linearly independent choices plus any number of linear combinations. For a particular factor, then, the weight vector is chosen such that its unsystematic portion is minimized. For example, the weights for the zero beta portfolio, x_o, are the solution to minimizing $x_o'\psi x_o$ subject to $x_o'\ell = 1$ and $x_o'b_1 = x_o'b_2 = . . . = x_o'b_k = 0$, where the b_i are the loadings on the i^{th} factor. This program is then repeated for each factor. More generally letting c_i be the standard unit vector, the weights for the k + 1 portfolios, χ, are the solutions to

$$\min_{x_i} \quad x_i'\psi x_i \quad , \quad i = 0, . . ., k$$

$$\text{s.t.} \quad x_i'A = c_i .$$

The solution is found by introducing k + 1 Lagrange multipliers and minimizing

$$L(x_i, \lambda_i) = x_i\psi x_i - 2\lambda_i\ (x_i'A - c_i), \tag{3-25}$$

where λ_i is the row vector of the multipliers. Thus

$$L(x_i,\lambda_i)\ \partial x_i = 0 => \quad \psi x_i - A = 0. \tag{3-26}$$

$$L(x_i\ \lambda_i)/\partial\lambda_i = 0 => \quad x_i'A - c_i = 0. \tag{3-27}$$

Defining χ as $<x_i>$, Λ as $<\lambda_i>$ and I as the identity matrix $<c_i>$, the solutions for all k + 1 weight vectors can be written as

$$\psi\chi - A\Lambda = \underline{0} \tag{3-26a}$$

$$\chi'A - I = \underline{0}. \tag{3-27a}$$

Eliminating the multipliers and solving for χ yields

$$\chi = \psi^{-1}A(A'\psi^{-1}A)^{-1}. \tag{3-28}$$

Once the portfolio weights are obtained as in (3-28), the data from the base period are discarded. This is done to avoid the circularity inherent in using the same data to fit and test the model. In particular, it is desirable for testing purposes to have estimates with known distributions. For example, one of the tests performed by Roll and Ross (1980) consists of regressing mean returns cross-sectionally on the factor loadings. A standard t-test is used to evaluate the significance of the estimated risk premia. Clearly, any results obtained by such a procedure depend on the previously determined factor solution and especially on the number of factors determined to be significant. Such a procedure seems little different from determining a one factor solution and running such a regression, then obtaining a two-factor solution and repeating the regression, and so on until the k^{th} factor is insignificant. A discussion of the problems with such pre-test estimators may be found in Judge *et al*. (1980, pp. 54-94).

Once the weights are obtained, the returns on the arbitrage portfolios are calculated using the test period data. Letting R be the n x t matrix of individual security returns, the arbitrage portfolio returns are

$$R'\chi = R'\psi^{-1}A'(A'\psi^{-1}A)^{-1}. \tag{3-29}$$

Estimates obtained in this fashion have some interesting features. First, they are equivalent to the estimates obtained from running cross-sectional GLS regressions of the security returns on the factor loadings for each day, and the mean returns on the arbitrage portfolios are equivalent to regression coefficients of mean security returns on the factor loadings. There is no question of "statistical" difference as considered by Oldfield and Rogalski (1981). Second, the estimated weights are actually just factor scoring coefficients estimated with Bartlett's approach, and are unbiased estimates of the true factors (Lawley and Maxwell (1971), p. 109). Bartlett's method also produces factor estimates that are univocal, i.e., uncorrelated with the other factors (Harman (1976), p. 385). Finally, the use of the inverse of the residual variances in (3-29) has the effect of correcting for the heteroscedasticity of the residuals and, all other things being equal, places greater weight on those securities with greater common variances.

The final step in estimating the arbitrage model is to use the factor risk premia as independent variables in time-series regressions of the form

$$\tilde{r}_{jt} = \hat{\alpha} + \hat{b}_{jo}\tilde{\lambda}_{o} + \hat{b}_{ji}\tilde{\lambda}_{1} + \ldots + \hat{b}_{jk}\tilde{\lambda}_{k} + \tilde{e}_{jt}, \qquad (3\text{-}30)$$

for each security. Equation (3-30) is the basis for the tests of the next chapter and the event study comparison with the market model in Chapter V.

Summary

In this chapter, the theory and estimating procedure underlying the arbitrage model were discussed. The result is a model in which security returns are linearly related to a set of unspecified, though measurable, latent variables. The next step is to compare the empirical results obtained from its use with the predictions of the underlying theory. To the extent that they are not in accord, the model loses one of its prime justifications. The last step in developing the arbitrage model is of a more practical nature. Theoretical consideration aside, the arbitrage model is somewhat more involved than other models (and more expensive to use), so its performance relative to simpler models is a subject worth investigating.

CHAPTER IV
TESTING THE ARBITRAGE THEORY

Introduction

In this chapter, attention turns to the empirical issues of the arbitrage model. Previous chapters have addressed the need for a multifactor model and examined evidence suggesting its appropriateness. In the last chapter, an arbitrage model was specified as an empirical analog to the APT. Because the content of the model stems directly from the predictions of the arbitrage theory, it becomes a natural vehicle for establishing the general validity of the APT.

For all of its simplicity and intuitive appeal, the arbitrage theory is rather limited; all that is indicated is that the return generating process has an approximate linear dimension less than the number of risky assets in the economy. It is important to realize, however, that the dimension reduction is the content of the theory, not the factors themselves. If the dimension of the structure of security returns is known, the factors are implicit in that structure. In this sense, they are best viewed as continuous versions of Arrow-Debreu pure or "primitive" securities. In the continuously distributed case, the number of states is equal to the number of securities; hence, n linearly independent securities are necessary to exactly span the state space. The arbitrage theory amounts to the assertion that the state space is approximately spanned by k + 1 linearly independent vectors, and that the degree of approximation improves as the number of securities increases. Interest in the theory, then, should not stem from the possibility of interpreting the factors as some collection of macro-variates. In fact, there is nothing in the theory which suggests that such an interpretation exists, certainly the theory does not require it.

The APT is not a causal theory; security returns are merely associated with some collection of measurements. These measurements are best interpreted as indices, the level of which have no intrinsic meaning. The best that can be said is that the indices are aggregate measures of the information sets and expectations of market participants. Changes in the levels of the factors are consequences of the arrival and assimilation of information and its role in the formulation of market expectations, a process about which little is known.

While the theory is mute in regard to the nature and number of the factors, estimates derived therefrom must have certain properties if the theory is to be empirically testable or practically usuable. Formally, all that is required from the theory is that

$$\lim_{n\to\infty} \frac{k}{n} = 0. \qquad (4\text{-}1)$$

Thus, for any finite collection of assets, the number of factors can be quite large, and that number can change as assets are added. In the non-stable case, where the number of factors and the factor loadings change as assets are added, the theory is probably of limited practical interest. What is hoped instead is that the economy has a fixed, finite number of sources of risk and that $k \ll n$. The practical interest in the theory in this light is inversely related to the ratio in (4-1), at least for a particular market.

The arguments above suggest that the practical interest in the APT is related to the degree of empirical parsimony possible through its use.

This subject is of interest in its own right and is taken up in the next section. Once the dimension of the model is fixed, subsequent sections report various univariate results obtained from the model, and, finally, a multivariate test of the APT. While a fixed dimension and relative stability of the factor loadings across securities are sufficient for research in this area to be interesting, one additional requirement is that the estimates contain a degree of intertemporal stability sufficient to justify their use vis-a-vis other, simpler models. Evidence relating to this stability is a by-product of the results of this chapter; however, a discussion of the implications is deferred to Chapter V.

Factor Analysis of Daily Security Returns

The purpose of this section is to arrive at an estimate of the number of relevant factors. The basic data consist of daily holding period returns including dividends on nearly 5000 New York Stock Exchange and American Stock Exchange listed securities extracted from the Center for Research in Security Prices (CRSP) (1983) daily returns file. The computations are performed on the University of Georgia IBM 370 (MVS/OS) using the maximum likelihood factor routine in the Statistical Analysis System (1982), 1982b version.

The first analysis performed is similar to that of Roll and Ross (1980). Returns for the first 1250 trading days in the CRSP file (7/3/62 - 6/19/67) were assembled for the first 30 securities (alphabetically) with a complete return series over the period. A maximum likelihood solution was obtained for one factor, two factors, and so on up to eight factors. The results are summarized in Table 4-1.

TABLE 4-1

SUMMARY INFORMATION

Number of Securities: 30

Number of Observations: 1250

Sample Period: 7/3/62 - 6/19/67

FACTOR ANALYSIS RESULTS

Number of Factors	Computed χ^2 Value	Degrees of Freedom	Probability K Factors Sufficient	Schwarz's Bayesian Criterion	Akaike's Information Criterion
1	526.78	405	<.0001	479.98	652.10
2	447.97	376	.0063	543.69	630.74
3*	371.70	348	.1830	605.09	609.86
4*	321.38	321	.4836	675.99	613.15
5*	282.11	295	.6953	748.92	625.58
6*	243.94	270	.8709	818.80	637.07
7*+	214.85	246	.9248	889.70	655.73
8*+	187.45	223	.9600	957.87	674.06

TABLE 4-1 (continued)

COMMUNALITY ESTIMATES

Number of Factors	Total Estimated Communality (Percent)	Percentage of Total Estimated Communality Attributable to Each Factor							
		1	2	3	4	5	6	7	8
1	17.60	100							
2	19.57	91.47	8.53						
3*	19.34	88.24	11.76						
4*	21.11	81.34	10.70	7.96					
5*	22.44	76.81	9.94	7.40	5.84				
6*	21.92	70.97	12.21	9.31	7.51				
7*	20.73	64.27	15.46	10.53	9.75				
8*	83.39	56.66	19.47	13.37	5.89	1.87			

SQUARED CANONICAL CORRELATIONS

Number of Factors	Squared Canonical Correlations for Each Factor with the Variables (Percent)							
	1	2	3	4	5	6	7	8
1	84.08							
2	84.30	33.37						
3*	100.00	78.96	33.33					
4*	100.00	79.40	33.65	27.38				
5*	100.00	79.69	33.67	27.44	22.98			
6*	100.00	100.00	73.78	32.62	26.95	22.94		
7*	100.00	100.00	100.00	65.70	31.54	23.88	22.52	
8*	100.00	92.79	81.56	75.24	57.25	29.81	26.34	22.93

* Indicates a Heywood Case

+ Indicates a lack of convergence after 15 iterations, convergence is approximate.

Several interesting results emerge from this analysis. First, at the 5 percent level, the hypothesis that three factors are sufficient cannot be rejected (Table 4-1). Akaike's criterion reaches its minimum at three as well, but Schwarz's criterion only picks up the dominant first factor. The three factor result agrees with that ultimately obtained by Roll and Ross (1980). A problem that can arise with the algorithm employed is the potential for Heywood cases. A Heywood case occurs when the factor model is a perfect fit for one or more of the variables (i.e., a communality of one). The likelihood function is discontinuous at such points. The solution adopted here is to delete the offending variable(s) and fit k - 1 (or k minus the number of eliminated variables) factors. Heywood cases arise from the numerical algorithm; at each iteration, the securities are weighted by the reciprocal of their uniqueness. Variables with greater communalities are thus given greater weight. If there are too many factors relative to the number of variables, the uniqueness can approach zero, assigning an extremely large weight to a particular security. For example, in the three factor solution, the communality for one of the securities moves from .49 to 1.00 in five iterations. It is removed from the sample and convergence is established in two additional iterations, where covergence requires that no changes in the communality estimates exceed .001 in absolute magnitude. Once the variable is deleted, two factors are fit to the remaining 29 variables.

The communality estimates in Table 4-1 refer to the variance explained by the factors collectively and individually. They are obtained in the following way: the hypothetical population correlation matrix is reproduced using k factors after weighting each variable by the reciprocal of its uniqueness. The percentage of variance explained by the factors is just the trace of reproduced correlation matrix divided by the trace of the sample correlation matrix (which is simply the number of variables). So, in the two factor solution, an estimated 19.57 percent of the total variance is systematic. Of that systematic portion, the first factor accounts for 91.47 percent with the second accounting for the remaining 8.53 percent. Information regarding the adequacy of the sample size is contained in Table 4-1. The squared canonical correlations for each factor with the variables are measures of the extent to which the factors can be predicted from the variables and can be interpreted as squared multiple correlation coefficients. Inspecting Table 4-1, the first factor can be predicted with reasonable accuracy (R^2 = 88.24%), but there are insufficient variables to accurately measure the others. The perfect correlations stem from the Heywood cases. The Heywood cases and low multiple correlations for factors beyond the first indicate a need for a larger sample.

In an attempt to improve the results, the sample size was doubled to 60 securities, selected sequentially from the CRSP file beginning with the thirty-first security with complete returns data. The results are reported in Table 4-2.

With 60 securities, the results are not drastically different from those obtained with the smaller sample. Again, Schwarz's criterion points to the single dominant factor. Based on Akaike's criterion, a five factor solution is optimal, though only slightly better than a four factor representation, and, depending on the significance level chosen, the maximum likelihood criterion would also indicate five factors at the most. The communality estimates obtained with the five factor solution are quite similar to those obtained with three factors in the smaller sample, indicating again that about 20 percent of the total variance is systematic.

TABLE 4-2

SUMMARY INFORMATION

Number of Securities: 60

Number of Observations: 1250

Sample Period: 7/3/62 - 6/19/67

FACTOR ANALYSIS RESULTS

Number of Factors	Computed χ^2 Value	Degrees of Freedom	Probability K Factors Sufficient	Schwarz's Bayesian Criterion	Akaike's Information Criterion
1	2235.56	1710	<.0001	1566.12	2516.54
2	1920.02	1651	<.0001	1616.36	2314.28
3	1779.84	1593	.0007	1752.23	2288.44
4	1638.73	1536	.0339	1883.99	2259.50
5	1522.11	1480	.2180	2024.60	2253.39
6	1432.15$^+$	1425	.4419	2175.20	2272.38
7	1344.32$^+$	1371	.6915	2323.26	2291.43
8	1264.53	1318	.8515	2471.82	2316.62

TABLE 4-2 (continued)

COMMUNALITY ESTIMATES

Number of Factors	Total Estimated Communality (Percent)	Percentage of Total Estimated Communality Attributable to Each Factor							
		1	2	3	4	5	6	7	8
1	14.25	100.00							
2	16.27	90.17	9.83						
3	17.47	85.12	9.35	5.53					
4	18.66	80.56	8.91	5.35	5.18				
5	19.86	76.81	8.58	5.22	4.96	4.43			
6+	21.66	72.25	8.41	6.79	4.58	4.43	3.56		
7+	22.20	70.47	8.02	5.62	4.56	4.41	3.53	3.40	
8	22.99	68.37	7.83	5.06	4.60	4.27	3.41	3.38	3.08

SQUARED CANONICAL CORRELATIONS

Number of Factors	Squared Canonical Correlations for Each Factor with the Variables (Percent)							
	1	2	3	4	5	6	7	8
1	89.53							
2	89.80	48.98						
3	89.92	49.50	36.69					
4	90.02	49.96	37.45	36.70				
5	90.15	50.56	38.35	37.13	34.55			
6+	90.37	52.21	46.86	37.31	36.54	31.62		
7+	90.37	51.64	42.80	37.77	37.01	31.96	31.15	
8	90.41	51.93	41.10	38.31	37.07	32.01	31.80	29.79

+Indicates a lack of convergence after 15 iterations, convergence is approximate.

As indicated in Table 4-2, there is a significant increase in the squared canonical correlation for the second factor, rising from about 33 percent to 50 percent. The increased sample size also eliminates the troublesome Heywood cases, with no communalities exceeding .50. The algorithm converges rapidly for up to five factors, generally requiring no more than five iterations. Although convergence was achieved for the eight factor solution (fifteen iterations), the six and seven factor attempts did not converge. The conclusion from this analysis is that no more than five factors are needed to account for the systematic intercorrelation for sixty securities. These analyses were repeated for several different samples with the same general results: not more than five factors are needed.

As a final check, the two samples were combined with ten other securities for a total of one hundred. The results in Table 4-3 indicate that the five factor solution originally indicated is insufficient.

As indicated in Table 4-3, factor solutions for up to six factors converge rapidly (as few as three iterations); however, the maximum likelihood criterion indicates that even a seven factor solution is inadequate. With eight or nine factors, the algorithm was unable to find even an approximate solution. Since Horn and Engstrom's (1979) results indicate the use of a larger significance level with the larger sample, an eleven or even ten factor solution is indicated. Akaike's criterion indicates six factors with Schwarz's criterion again identifying only the dominant factor. The use of the larger sample results in general improvement in the predictability of the first several factors; however, the communality totals are relatively unchanged.

The results from the different sized samples contain some similarities. First, the Bayesian criterion always indicates a single factor.

TABLE 4-3
SUMMARY INFORMATION

Number of Securities: 100

Number of Observations: 1250

Sample Period: 7/3/62 - 6/19/67

FACTOR ANALYSIS RESULTS

Number of Factors	Computed χ^2 Value	Degrees of Freedom	Probability K Factors Sufficient	Schwarz's Bayesian Criterion	Akaike's Information Criterion
1	6709.35	4850	<.0001	4166.77	7307.36
2	6141.14	4751	<.0001	4228.99	6923.85
3	5720.68	4653	<.0001	4363.48	6689.99
4	5368.16	4556	<.0001	4529.18	6523.71
5	5131.87	4460	<.0001	4751.09	6473.95
6	4943.11	4365	<.0001	4993.83	6473.00
7+	4766.57	4271	<.0001	5239.22	6481.47
**					
10+	4276.14	3995	.001	5973.65	6534.20
11	4141.46	3905	.0043	6226.05	6577.22
12+	3992.99	3816	.0226	6467.67	6603.82
13*+	3843.42	3728	.0916	6705.07	6627.09

TABLE 4-3 (continued)

COMMUNALITY ESTIMATES

Number of Factors	Total Estimated Communality (Percent)	Percentage of Estimated Communality Attributable to Each Factor												
		1	2	3	4	5	6	7	8	9	10	11	12	13
1	15.44	100.00												
2	17.08	92.17	7.83											
3	18.75	85.79	7.46	6.76										
4	20.19	81.02	7.41	6.53	5.03									
5	21.18	77.92	7.12	6.30	4.98	3.68								
6	22.04	75.53	6.87	6.09	4.84	3.57	3.10							
7+	23.92	71.32	9.06	5.84	4.65	3.34	2.91	2.89						
**														
10+	26.24	66.34	7.61	5.50	4.35	3.19	2.94	2.79	2.53	2.44	2.31			
11	26.94	64.77	7.26	5.40	4.24	3.17	2.86	2.73	2.59	2.44	2.28	2.27		
12+	27.87	63.13	7.02	5.27	4.15	3.11	3.01	2.67	2.55	2.43	2.26	2.24	2.17	
13*+	23.56	56.35	8.97	6.18	4.91	3.73	3.25	3.10	3.01	2.70	2.70	2.65	2.46	

TABLE 4-3 (continued)

SQUARED CANONICAL CORRELATIONS

Number of Factors	Squared Canonical Correlations for Each Factor with the Variables (Percent)												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	93.92												
2	94.03	57.23											
3	94.15	58.30	55.89										
4	94.24	59.95	56.88	50.38									
5	94.29	60.12	57.17	51.34	43.81								
6	94.33	60.22	57.32	51.61	44.03	40.56							
7+	94.46	68.44	58.29	52.64	44.39	41.00	40.86						
**													
10+	94.57	66.62	59.06	53.28	45.57	43.50	42.26	40.00	39.05	37.78			
11	94.58	66.16	59.27	53.32	46.04	43.52	42.41	41.08	40.00	38.07	37.90		
12+	94.62	66.17	59.48	53.61	46.47	45.59	42.67	41.53	40.34	38.64	38.47	37.64	
13*+	100.00	92.99	67.88	59.29	53.61	46.77	43.33	42.17	41.50	38.92	38.85	38.40	36.71

* Indicates a Heywood Case
+ Indicates exact convergence not achieved, results are approximate
** Indicates a lack of approximate convergence

Based on the description in Chapter III, this is not surprising; all other things equal, large numbers of observations (1250 in this case) will tend to greatly reduce the number of significant factors in this approach. The classical maximum likelihood estimate behaves in a relatively predictable fashion as well with the number of factors determined to be significant varying with the sample size. In fact, the number of factors appears to be roughly proportional to the number of securities at 8 to 10 percent. Akaike's criterion is relatively stable, indicating five factors with sixty securities and five to six with one hundred.

An important similarity between the results is the behavior of the estimated total communality; one conclusion which can be drawn is that the portion of daily security returns which is systematic is approximately 20 percent. Of this portion, the market factor accounts for about 75 percent or 15 percent of the total variance. If the one factor solution is viewed as the best possible single index, then the inclusion of multiple indices can result in a 33 percent (.05/.15) improvement in systematic risk estimates. Moreover, inspection of the factor loadings indicates that this potential gain is not uniformly spread among the securities; in some cases, the variance explained by factors beyond the first is greater than the single factor explanatory power--evidence of significant "extra-market" sources of risk.

As a final attempt at objectively determining the approximate dimension of the model, the analysis of the one hundred securities was extended to include an additional five hundred observations per security. The results were virtually identical with those obtained in the previous trial and are not reported here. The very similar results stem from the fact that there is little difference in the sample correlation matrices despite the different lengths of time.

When the sample size is increased to 150 securities, the computational requirements become prohibitive, with a six factor solution requiring more than forty minutes of CPU time. In comparison, the six factor solution with 100 securities required only twelve minutes.

This section began in an attempt to arrive at an objective determination of the number of factors. Based on analyses of different sample sizes, only one criterion is completely consistent regarding the number of factors. Schwarz's Bayesian criterion indicates that the basic intuition of the market model is correct; namely, there is a single dominant factor present in security returns. On the other hand, the communality estimates indicate significant gains to be had from additional information, particularly for certain securities. The fact that the variance explained by factors beyond the first is concentrated in a subset of the securities improves the case for a multi-index approach. If the securities in the subset are relatively homogeneous, then single-index models may systematically mis-price their riskiness. This potential bias could account for anomalies associated with the market model. Some evidence for the validity of this speculation is offered in Chapter V.

For the reasons discussed above, the dimension of the arbitrage model must be specified subjectively. The decision was made to continue this research using six factors, the number indicated by Akaike's criterion. This choice also agrees with the five extra-market clusters identified by Arnott (1980). If anything, six factors would appear to be more than enough. Referring to Table 4-3, factors beyond the fourth individually account for less than one percent of the total variation and less than five percent of the estimated communality.

Other techniques are available for arriving at a decision in regard to the number of factors. The two most popular are Cattell's (1966) scree chart and Kaiser's (1960) eigenvalue criterion. Both approaches are based on the relative magnitudes of the eigenvalues of the sample correlation matrix. Cattell's method amounts to inspecting a chart of the eigenvalues plotted in order of decreasing magnitude and looking for breaks in the pattern. Because the average eigenvalue of a correlation matrix is unity, Kaiser's rule is to retain all eigenvalues with values greater than one. Neither of these approaches is especially enlightening for the securities data; the only clear break in the ordered eigenvalues occurs at the second factor and the number of eigenvalues exceeding one is quite large (23 out of 100 in one case).

In summary, there is sufficient noise in the ex post realized returns that it is impossible to objectively determine the number of factors beyond the first. Grouping techniques would no doubt reduce the noise content, but the results of this section indicate that randomly formed portfolios would probably swamp the extra-market components. It appears likely that the number of factors present ex ante is less than six, and evidence for this will be forthcoming in the tests of the APT. In the next section, the time periods and sampling techniques used in this study are described along with some properties of the six factor solutions in different samples.

Preliminary Analyses of the Arbitrage Model

The basic data used to test the APT once again consist of daily security returns from the CRSP file. Three non-overlapping base periods were chosen for the factor analyses, each covering 1250 trading days (about five years). Within each of three base periods, three samples of 100 securities each were created by taking every tenth security with a complete returns series from the CRSP file beginning with the first, second and third listed securities. Interval sampling was used to avoid undue concentration in certain industries; for example, alphabetic selection would result in one group being dominated by financial institutions--over thirty have names beginning with the letters of "Fi." To avoid confusing the samples, the notational convention of designating the time periods as 1, 2, and 3 and the samples as a, b, and c is adopted; thus, sample 2b is the second sample in the second time period. The exact dates for the base periods are

Base Period 1: 7/3/62 - 6/19/67
Base Period 2: 7/20/67 - 7/11/72
Base Period 3: 7/12/72 - 6/23/77.

Once the data were obtained, a six factor maximum likelihood solution was obtained for each of the nine groups. Table 4-4 summarizes the results. The total communality estimates in the first two base periods are all between 20 and 25 percent; however, the third period totals are greater; between 26 and 32 percent of the total variance is systematic. The market factor accounts for 71 to 80 percent of the total communality, with increased importance in the third period.

As discussed in Chapter III, any orthogonal linear transformation of the initial solution generates a mathematically equivalent result. Because the APT is designed to explain cross-sectional differences in security returns, a weighted Varimax rotation (Cureton and Mulaik, 1975) was used to increase the cross-sectional variation in the factor loadings. With this rotation, the factor loadings for the individual securities are first weighted by reciprocal of their uniqueness estimates; then, an orthogonal transformation matrix is determined such that the variance of the loadings on a particular factor is maximized. By maximizing the variance of the column loadings, the larger estimates are increased and the smaller estimates decreased. The weighted Varimax rotation also

TABLE 4-4
SUMMARY INFORMATION

Base Period 1: 7/3/62 - 6/19/67
Base Period 2: 6/20/67 - 7/11/72
Base Period 3: 7/12/72 - 6/23/77
Trading Days per Period: 1250
Samples per period: 3
Number of Securities per Sample: 100

INITIAL SOLUTION

	BASE PERIOD 1			BASE PERIOD 2			BASE PERIOD 3		
SAMPLE	a	b	c	a	b	c	a	b	c
TOTAL COMMUNALITY	22.05	24.10	21.41	21.13	22.45	21.90	26.10	31.75	28.07
PERCENTAGE OF TOTAL FOR EACH FACTOR	a	b	c	a	b	c	a	b	c
1	70.97	76.65	75.41	76.44	77.15	75.58	78.37	78.48	79.48
2	10.00	7.77	7.94	8.05	6.75	9.16	7.23	7.43	6.87
3	7.44	5.18	5.67	4.59	5.47	4.69	4.36	4.87	4.16
4	4.27	3.79	4.53	4.05	3.95	4.01	4.07	3.36	3.58
5	3.86	3.52	3.44	3.51	3.44	3.41	3.11	3.19	3.19
6	3.46	3.09	3.02	3.36	3.25	3.15	2.86	2.67	2.72

increases the "gain" on factors two through six at the expense of the first factor. As indicated in Table 4-5, the total communality is spread fairly evenly over the first four factors as a result. This effect should make it possible to measure the factors beyond the first with greater accuracy.

The three test periods in this study are, for each sample, the five hundred trading days subsequent to the base periods. The exact dates are

Test Period 1: 6/20/67 - 7/25/69

Test Period 2: 7/12/72 - 7/08/74

Test Period 3: 6/24/77 - 6/18/79.

To calculate the returns on the arbitrage portfolios, the weights are obtained using eq. (3-28)

$$X = \Psi^{-1}A(A\Psi^{-1}A)^{-1}, \tag{3-28}$$

where A is the augmented factor loading matrix and Ψ is the diagonal matrix of residual variances. The arbitrage portfolio returns over the test periods are calculated using eq. (3-29)

$$R'X = R'\Psi^{-1}A(A\Psi^{-1}A)^{-1}, \tag{3-29}$$

where R is the 100 by 500 matrix of daily returns. As noted in Chapter III, the arbitrage portfolio returns calculated in this fashion are identical to a time-series of coefficients obtained from 500 GLS cross-sectional regressions, with the zero beta return as the time series of estimated intercept terms. Summary univariate statistics are reported for the nine samples in Tables 4-6 through 4-14.

In the first test period, covering 7/67 - 7/69, the zero beta return is insignificantly different from zero in all three samples. Point estimates of the average daily return range from a negative 2.6 basis points to a positive 3 basis points, with standard deviations of 60 to 70 basis points. Averaging the three point estimates and annualizing the results,

TABLE 4-5
VARIMAX ROTATED SOLUTION

	BASE PERIOD 1			BASE PERIOD 2			BASE PERIOD 3		
SAMPLE	a	b	c	a	b	c	a	b	c
TOTAL COMMUNALITY	22.05	24.10	21.41	21.13	22.45	21.90	26.10	31.75	28.07
PERCENTAGE OF TOTAL FOR EACH FACTOR	a	b	c	a	b	c	a	b	c
1	17.62	32.99	26.94	26.11	22.94	33.65	23.31	29.64	27.53
2	17.20	27.18	19.23	25.03	20.23	29.14	21.03	24.88	23.16
3	21.37	15.00	20.06	19.73	16.42	19.73	19.91	18.79	15.02
4	19.42	13.64	14.21	16.95	19.55	9.35	17.78	14.24	15.34
5	14.57	6.12	15.04	7.40	12.77	4.80	12.07	8.50	10.50
6	9.82	5.07	4.50	4.78	8.06	3.34	5.90	3.95	8.46

a return in the neighborhood of 1 percent is indicated, at a time when money market rates varied from four to six percent (Federal Reserve Bulletin, 1970). In the second test period (7/72 - 6/74), the zero beta returns are larger and vary from 2 to 7 basis points per day. An approximate annual yield of 18 percent is obtained by averaging the three. Once again, the standard deviations of the estimates are large relative to the point values. Also, the standard deviations are similar to each other, ranging from 89 to 106 basis points. A low estimate of .4 basis points per day was obtained in the third period (7/77 - 6/79) with a high estimate of 5 points. The average of the three is about 2 points per day or a 7.2 percent annual yield which compares favorably with the 7.19 percent yield on 90-day Treasury Bills for the year 1978 (Federal Reserve Bulletin, 1979). The standard deviations range from 49 to 63 basis points. Because of the variability in the zero beta return, the estimates are not especially reliable; however, the average estimates obtained in the first and third periods are fairly close to the then prevailing interest rates. One problem with the GLS estimates as opposed to OLS estimates is that unbiasedness is achieved at the expense of greater variance in the estimated factor scores. When the zero beta returns are estimated using OLS, the averages in basis points per day for the three periods are 2.2, 3.53, and 1.47 respectively. The estimate in the first period is quite close to the 1968 daily return on 90-day Treasury Bills which averaged 2.18 points per day (Federal Reserve Bulletin, 1970). The estimate in the second period is still high, though less so than with the GLS estimate, and the third period estimate appears low. It is difficult to generalize from these results; however, there appears to be no tendency for the estimates to be consistently higher than some measure of the risk-free rate, unlike

TABLE 4-6
SUMMARY INFORMATION

Test Period: 1a
Number of Trading Days: 500
Number of Securities: 100

UNIVARIATE STATISTICS FOR TEST PERIOD

Portfolio	Average Daily Return	Standard Deviation	Computed T Statistic	Significance Level
Zero Beta	.00006	.0067	.199	.842
Arbitrage #1	.1128	2.0702	1.219	.224
Arbitrage #2	.0317	1.2539	.566	.572
Arbitrage #3	-.1503	1.0789	-3.115	.002
Arbitrage #4	.0223	1.1348	.440	.660
Arbitrage #5	-.0120	1.3648	- .197	.844
Arbitrage #6	.0577	1.3576	.950	.343

CORRELATIONS AMONG ARBITRAGE PORTFOLIO RETURNS

Portfolio	Zero Beta	Arbitrage #1	Arbitrage #2	Arbitrage #3	Arbitrage #4	Arbitrage #5	Arbitrage #6
Zero Beta	1.000						
Arbitrage #1	- .750	1.000					
Arbitrage #2	- .190	.109	1.000				
Arbitrage #3	.074	- .036	- .280	1.000			
Arbitrage #4	- .084	.006	- .109	- .044	1.000		
Arbitrage #5	- .192	- .002	- .166	- .201	.041	1.000	
Arbitrage #6	- .160	.220	.152	- .141	.111	- .175	1.000

TABLE 4-7
SUMMARY INFORMATION

Test Period: 1b
Number of Trading Days: 500
Number of Securities: 100

UNIVARIATE STATISTICS FOR TEST PERIOD

Portfolio	Average Daily Return	Standard Deviation	Computed T Statistic	Significance Level
Zero Beta	-.00026	.0061	- .944	.345
Arbitrage #1	-.1180	.9780	-2.698	.007
Arbitrage #2	.1947	2.0155	2.160	.031
Arbitrage #3	.0143	1.3830	.232	.818
Arbitrage #4	.0232	1.0805	.480	.631
Arbitrage #5	.0186	1.3815	.300	.764
Arbitrage #6	.0933	1.6222	1.286	.199

CORRELATIONS AMONG ARBITRAGE PORTFOLIO RETURNS

Portfolio	Zero Beta	Arbitrage #1	Arbitrage #2	Arbitrage #3	Arbitrage #4	Arbitrage #5	Arbitrage #6
Zero Beta	1.000						
Arbitrage #1	.265	1.000					
Arbitrage #2	- .765	- .301	1.000				
Arbitrage #3	- .347	- .058	.233	1.000			
Arbitrage #4	- .171	- .255	.090	- .059	1.000		
Arbitrage #5	.116	- .075	- .191	- .007	- .041	1.000	
Arbitrage #6	- .308	- .113	.236	.223	- .209	- .112	1.000

TABLE 4-8
SUMMARY INFORMATION

Test Period: 1c
Number of Trading Days: 500
Number of Securities: 100

UNIVARIATE STATISTICS FOR TEST PERIOD

Portfolio	Average Daily Return	Standard Deviation	Computed T Statistic	Significance Level
Zero Beta	.0003	.0064	1.054	.293
Arbitrage #1	-.0787	1.0224	-1.721	.086
Arbitrage #2	.0566	1.7255	.734	.463
Arbitrage #3	.1169	1.5585	1.677	.094
Arbitrage #4	.0006	1.4533	.010	.992
Arbitrage #5	-.0873	1.3578	-1.440	.151
Arbitrage #6	-.0363	1.4333	- .566	.571

CORRELATIONS AMONG ARBITRAGE PORTFOLIO RETURNS

Portfolio	Zero Beta	Arbitrage #1	Arbitrage #2	Arbitrage #3	Arbitrage #4	Arbitrage #5	Arbitrage #6
Zero Beta	1.000						
Arbitrage #1	.351	1.000					
Arbitrage #2	- .623	- .310	1.000				
Arbitrage #3	- .039	- .003	- .013	1.000			
Arbitrage #4	- .312	- .221	.103	.001	1.000		
Arbitrage #5	- .324	- .383	.161	- .406	.101	1.000	
Arbitrage #6	- .138	- .070	.160	- .168	- .108	.098	1.000

TABLE 4-9
SUMMARY INFORMATION

Test Period: 2a
Number of Trading Days: 500
Number of Securities: 100

UNIVARIATE STATISTICS FOR TEST PERIOD

Portfolio	Average Daily Return	Standard Deviation	Computed T Statistic	Significance Level
Zero Beta	.0002	.0106	.465	.642
Arbitrage #1	.1659	1.9190	1.933	.054
Arbitrage #2	-.0016	1.5070	- .024	.981
Arbitrage #3	.0110	1.4200	.173	.863
Arbitrage #4	.0175	1.2895	.303	.762
Arbitrage #5	.0271	1.5663	.386	.699
Arbitrage #6	.0345	1.5421	.501	.617

CORRELATIONS AMONG ARBITRAGE PORTFOLIO RETURNS

Portfolio	Zero Beta	Arbitrage #1	Arbitrage #2	Arbitrage #3	Arbitrage #4	Arbitrage #5	Arbitrage #6
Zero Beta	1.000						
Arbitrage #1	- .695	1.000					
Arbitrage #2	- .327	.270	1.000				
Arbitrage #3	.370	- .301	- .136	1.000			
Arbitrage #4	.082	- .101	- .151	- .233	1.000		
Arbitrage #5	- .168	- .014	- .050	.002	- .086	1.000	
Arbitrage #6	- .066	- .089	- .047	- .088	.062	- .040	1.000

TABLE 4-10
SUMMARY INFORMATION

Test Period: 2b
Number of Trading Days: 500
Number of Securities: 100

UNIVARIATE STATISTICS FOR TEST PERIOD

Portfolio	Average Daily Return	Standard Deviation	Computed T Statistic	Significance Level
Zero Beta	.0006	.0095	1.426	.154
Arbitrage #1	.0079	1.3987	.126	.900
Arbitrage #2	.1534	1.5485	2.215	.027
Arbitrage #3	.0191	1.3076	.327	.744
Arbitrage #4	-.0111	1.3230	- .188	.851
Arbitrage #5	-.0421	1.5497	- .607	.544
Arbitrage #6	.0136	1.2597	.241	.810

CORRELATIONS AMONG ARBITRAGE PORTFOLIO RETURNS

Portfolio	Zero Beta	Arbitrage #1	Arbitrage #2	Arbitrage #3	Arbitrage #4	Arbitrage #5	Arbitrage #6
Zero Beta	1.000						
Arbitrage #1	- .438	1.000					
Arbitrage #2	- .417	.113	1.000				
Arbitrage #3	.144	- .262	- .154	1.000			
Arbitrage #4	- .088	- .002	- .053	- .319	1.000		
Arbitrage #5	.329	- .042	- .074	- 1.75	.039	1.000	
Arbitrage #6	.192	- .250	- .077	- 1.82	- .075	.119	1.000

TABLE 4-11
SUMMARY INFORMATION

Test Period: 2c
Number of Trading Days: 500
Number of Securities: 100

UNIVARIATE STATISTICS FOR TEST PERIOD

Portfolio	Average Daily Return	Standard Deviation	Computed T Statistic	Significance Level
Zero Beta	.0007	.0089	1.786	.075
Arbitrage #1	.0953	1.4644	1.455	.146
Arbitrage #2	.0626	1.3501	1.037	.300
Arbitrage #3	.0138	1.0822	.284	.776
Arbitrage #4	-.0893	1.5654	-1.275	.203
Arbitrage #5	.0647	1.5594	- .928	.354
Arbitrage #6	-.0674	1.6286	- .925	.355

CORRELATIONS AMONG ARBITRAGE PORTFOLIO RETURNS

Portfolio	Zero Beta	Arbitrage #1	Arbitrage #2	Arbitrage #3	Arbitrage #4	Arbitrage #5	Arbitrage #6
Zero Beta	1.000						
Arbitrage #1	- .219	1.000					
Arbitrage #2	- .123	- .285	1.000				
Arbitrage #3	.108	.107	- .399	1.000			
Arbitrage #4	- .387	- .047	- .090	.112	1.000		
Arbitrage #5	- .219	.033	- .095	.192	.085	1.000	
Arbitrage #6	.048	- .025	- .336	.149	.170	.1029	1.000

TABLE 4-12
SUMMARY INFORMATION

Test Period:	3a
Number of Trading Days:	500
Number of Securities:	100

UNIVARIATE STATISTICS FOR TEST PERIOD

Portfolio	Average Daily Return	Standard Deviation	Computed T Statistic	Significance Level
Zero Beta	.00007	.0063	.253	.800
Arbitrage #1	.0474	.9139	1.160	.247
Arbitrage #2	.1099	1.0036	2.450	.016
Arbitrage #3	-.0104	1.0073	- .217	.829
Arbitrage #4	-.0287	.9802	- .655	.513
Arbitrage #5	-.0053	.9601	- .123	.902
Arbitrage #6	-.0649	.9340	-1.550	.122

CORRELATIONS AMONG ARBITRAGE PORTFOLIO RETURNS

Portfolio	Zero Beta	Arbitrage #1	Arbitrage #2	Arbitrage #3	Arbitrage #4	Arbitrage #5	Arbitrage #6
Zero Beta	1.000						
Arbitrage #1	- .128	1.000					
Arbitrage #2	- .251	.094	1.000				
Arbitrage #3	- .737	- .020	.018	1.000			
Arbitrage #4	.302	- .053	- .125	- .263	1.000		
Arbitrage #5	- .291	.110	.114	.160	- .264	1.000	
Arbitrage #6	- .163	.005	- .102	.155	- .158	- .132	1.000

TABLE 4-13
SUMMARY INFORMATION

Test Period: 3b
Number of Trading Days: 500
Number of Securities: 100

UNIVARIATE STATISTICS FOR TEST PERIOD

Portfolio	Average Daily Return	Standard Deviation	Computed T Statistic	Significance Level
Zero Beta	.0005	.0068	1.535	.125
Arbitrage #1	.0843	1.0709	1.760	.079
Arbitrage #2	-.0087	.9648	- .201	.841
Arbitrage #3	-.0623	.9539	-1.460	.145
Arbitrage #4	-.0228	.9241	- .551	.582
Arbitrage #5	-.0104	.9786	- .239	.811
Arbitrage #6	-.0050	.9335	- .120	.905

CORRELATIONS AMONG ARBITRAGE PORTFOLIO RETURNS

Portfolio	Zero Beta	Arbitrage #1	Arbitrage #2	Arbitrage #3	Arbitrage #4	Arbitrage #5	Arbitrage #6
Zero Beta	1.000						
Arbitrage #1	- .189	1.000					
Arbitrage #2	.185	- .102	1.000				
Arbitrage #3	- .742	- .053	- .278	1.000			
Arbitrage #4	- .210	.123	- .035	.043	1.000		
Arbitrage #5	- .554	.035	- .084	.401	.044	1.000	
Arbitrage #6	- .100	- .014	.003	- .014	.018	- .130	1.000

TABLE 4-14
SUMMARY INFORMATION

Test Period: 3c
Number of Trading Days: 500
Number of Securities: 100

UNIVARIATE STATISTICS FOR TEST PERIOD

Portfolio	Average Daily Return	Standard Deviation	Computed T Statistic	Significance Level
Zero Beta	.0004	.0049	.181	.856
Arbitrage #1	.0812	.9404	1.930	.054
Arbitrage #2	.0132	.9073	.325	.745
Arbitrage #3	-.0329	1.0015	- .734	.463
Arbitrage #4	-.0059	.8027	- .164	.870
Arbitrage #5	.0296	.9363	.706	.480
Arbitrage #6	.0194	.9542	.454	.650

CORRELATIONS AMONG ARBITRAGE PORTFOLIO RETURNS

Portfolio	Zero Beta	Arbitrage #1	Arbitrage #2	Arbitrage #3	Arbitrage #4	Arbitrage #5	Arbitrage #6
Zero Beta	1.000						
Arbitrage #1	- .365	1.000					
Arbitrage #2	.109	- .177	1.000				
Arbitrage #3	- .224	- .208	- .020	1.000			
Arbitrage #4	.029	- .004	- .122	- .201	1.000		
Arbitrage #5	- .440	.120	- .085	.015	- .022	1.000	
Arbitrage #6	- .165	.098	.032	.167	- 1.46	.074	1.000

the same estimates reported in studies of the market model (e.g., Fama and MacBeth, 1973).

Also reported in Tables 4-6 through 4-14 are the estimated returns on the arbitrage portfolios and their standard deviations. In the first period, one portfolio in sample a has a significantly non-zero return while b and c each have two such portfolios (at a 10% level). Samples 2a and 2b each contain one significant return, and 2c contains none. In the third period, each sample has one portfolio with a statistically non-zero return.

In the base periods, the hypothetical factors were constructed such that they were mutually uncorrelated with unit variances. Estimates of the factor scores will in general possess neither property exactly. In the first two periods, the variances generally exceed one, while in the third period they are relatively close.

More troublesome are the correlations of the arbitrage portfolio returns with the zero beta return; in every case, there is at least one arbitrage portfolio with a substantial negative correlation. This unanticipated result is difficult to explain. However, if the risk premia are defined as excess returns above the zero beta return, then an inverse relationship implies that the nominal premia are not constant and decrease when the zero beta return increases. Alternatively, noise in the *ex post* data may give rise to differential measured sensitivities to the zero beta return. In this case, an extra factor may appear to exist and the appropriate dimension of the model would be over-estimated. The extra factor would not be priced, however. A purely empirical explanation is readily apparent when the structure of the arbitrage portfolios is examined. The portfolios with large negative correlations correspond to factors which tend to be dominated by public utilities, a group traditionally considered to be interest rate sensitive. This should not necessarily be interpreted as evidence for an interest rate factor. The utilities are regulated, they have substantial dividend yields, and they tend to be large. Thus, the phenomenon could relate to regulatory risk or lag, taxation of dividends, a size effect, or some other common characteristic. Whatever the explanation, the phenomenon is persistent and the collinearity between the zero beta return and the arbitrage returns may lead to econometric difficulty.

The results of this section indicate considerable volatility in the estimated zero beta returns and the risk premia, most of which are not statistically different from zero. It is important to emphasize here that no conclusions concerning the APT can be drawn at this point. The statistical significance (or lack thereof) of the risk premia cannot be interpreted as evidence for or against the APT for two reasons. First, on a theoretical level, such evidence relates to the linearity of the pricing relationship which for reasons discussed in Chapter III is not the relevant issue. Second, the risk premia are not uniquely determined empirically. The factor analysis uniquely determines the space into which returns are projected; the orientation of the factors within that space is only unique up to non-singular transformation. In practical terms, the significance of the risk premia are a function of the rotation chosen, using an oblique (non-orthogonal) rotation will generally result in greater numbers of "significant" factors at the expense of greater correlations among the risk premia. In fact, with the Promax rotation (Lawley and Maxwell, 1971), the degree of factor inter-correlation is to some extent controllable, and it can simply be increased until most of the premia are significant. This inherent indeterminateness renders attempts at identifying the number of relevant factors by cross-sectional regression meaningless. In sum, the requisite degree of linearity in the pricing relationship is unknown *a priori* and empirically indeterminate.

The result of a factor analysis is a unique estimate of a common factor space and the residual variances for the securities. Testing the APT requires examination of this unique information. Because the zero beta portfolio is constructed to be a member of the space perpendicular to the common factor space, it is unaffected by rotations of the factors within that space. When the additional requirement of minimum residual variance is imposed, the zero beta portfolio is uniquely defined. In fact, *in the hypothetical population*, the zero beta portfolio is the global minimum variance portfolio, and is thus mean-variance efficient.[1] The implications of this fact are pursued in the next section.

Univariate Results from the Arbitrage Model

In this section, the portfolio returns created in the previous section are used as independent variables in time-series regressions of the form specified in (3-30)

$$\tilde{r}_{jt} = \hat{\alpha}_j + \hat{b}_{jo}\tilde{\delta}_{ot} + \hat{b}_{j1}\tilde{\delta}_{1t} + \ldots + \hat{b}_{j6}\tilde{\delta}_{6t} + \tilde{e}_{jt}, \qquad (3\text{-}30)$$

$j = 1, \ldots, 100$

$t = 1, \ldots, 250$

$251, \ldots, 500.$

The intercept term is a measure of abnormal performance and will not differ from zero unless arbitrage opportunities exist. The coefficient of the zero beta return should be one, and the other coefficients should be generally significant. Mathematically, the coefficient of the zero beta return would be exactly equal to one if the testing were done in the base period using the actual residual covariance matrix to obtain the weights. In this case, the portfolio is mean-variance efficient in the sample and has an exactly zero correlation with any arbitrage portfolio. Because of this the estimated coefficient, $\hat{b}_{oj}$, is equal to the covariance of the return on security j with the zero beta return, divided by the variance of the zero beta return. It is easily verified (e.g., Roll, 1976) that the global minimum variance portfolio has the property that its covariance with any non-arbitrage portfolio is equal to its own variance; hence, an "estimated" coefficient would be unity with a zero error. As it is employed here, the zero beta portfolio return is an estimate of the unobservable minimum variance portfolio for all risky assets of the type under consideration.[2]

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[1] This fact is pointed out by Ingersoll (1982).

In the actual estimation, each of the three test periods is divided into two sub-periods, each of which covers 250 trading days. This was done in order to examine the stability of the estimates and the extent to which they deteriorate over time. The total number of estimated equations is 1800, consisting of nine samples of 100 securities each and two subperiods per sample. The results are reported in Tables 4-15 to 4-23.

In each of the tables, several items are tabulated. The first is the number of securities for which significant intercepts were found. The second and third are the number of estimates of $\hat{b}_o$ which differ from zero and one respectively. In the first 250 trading days of sample la, three of the 100 securities had significant intercepts, and 85 had significant coefficients on the zero beta portfolio, eight of which were significantly

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[2] This argument is extendable to all risky assets in the economy if it is possible to hedge any type of systematic risk using the subset of assets from which the factors were obtained.

TABLE 4-15

SUMMARY INFORMATION

Test Period: 1a

Number of Trading Days: 500

Number of Securities: 100

UNIVARIATE TESTS OF APT

First 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	3
$\hat{b}_0 = 0$	85
$\hat{b}_0 = 1$	8

Second 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	1
$\hat{b}_0 = 0$	88
$\hat{b}_0 = 1$	16

NUMBER OF EQUATIONS WITH k SIGNIFICANT FACTORS

(not including zero beta portfolio)

First 250 Trading Days

k	# of eq.
0	1
1	9
2	12
3	18
4	21
5	23
6	16

Second 250 Trading Days

k	# of eq.
0	2
1	13
2	20
3	14
4	22
5	17
6	12

Note: Significance level is .05 for two-tailed T-test.

TABLE 4-16

SUMMARY INFORMATION

Test Period: 1b

Number of Trading Days: 500

Number of Securities: 100

UNIVARIATE TESTS OF APT

First 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	2
$\hat{b}_0 = 0$	87
$\hat{b}_0 = 1$	11

Second 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	0
$\hat{b}_0 = 0$	90
$\hat{b}_0 = 1$	18

NUMBER OF EQUATIONS WITH k SIGNIFICANT FACTORS

(not including zero beta portfolio)

First 250 Trading Days

k	# of eq.
0	0
1	8
2	17
3	24
4	22
5	20
6	9

Second 250 Trading Days

k	# of eq.
0	1
1	11
2	20
3	25
4	17
5	19
6	7

Note: Significance level is .05 for two-tailed T-test.

TABLE 4-17

SUMMARY INFORMATION

Test Period: 1c

Number of Trading Days: 500

Number of Securities: 100

UNIVARIATE TESTS OF APT

First 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	3
$\hat{b}_o = 0$	91
$\hat{b}_o = 1$	12

Second 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	2
$\hat{b}_o = 0$	95
$\hat{b}_o = 1$	18

NUMBER OF EQUATIONS WITH k SIGNIFICANT FACTORS

(not including zero beta portfolio)

First 250 Trading Days

k	# of eq.
0	2
1	11
2	22
3	19
4	21
5	21
6	4

Second 250 Trading Days

k	# of eq.
0	0
1	4
2	23
3	26
4	23
5	18
6	6

Note: Significance level is .05 for two-tailed T-test.

TABLE 4-18

SUMMARY INFORMATION

Test Period: 2a

Number of Trading Days: 500

Number of Securities: 100

UNIVARIATE TESTS OF APT

First 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	0
$\hat{b}_0 = 0$	94
$\hat{b}_0 = 1$	24

Second 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	0
$\hat{b}_0 = 0$	89
$\hat{b}_0 = 1$	13

NUMBER OF EQUATIONS WITH k SIGNIFICANT FACTORS

(not including zero beta portfolio)

First 250 Trading Days

k	# of eq.
0	1
1	4
2	15
3	20
4	25
5	23
6	12

Second 250 Trading Days

k	# of eq.
0	1
1	2
2	13
3	18
4	28
5	22
6	16

Note: Significance level is .05 for two-tailed T-test.

TABLE 4-19

SUMMARY INFORMATION

Test Period: 2b

Number of Trading Days: 500

Number of Securities: 100

UNIVARIATE TESTS OF APT

First 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	0
$\hat{b}_0 = 0$	96
$\hat{b}_0 = 1$	13

Second 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	0
$\hat{b}_0 = 0$	88
$\hat{b}_0 = 1$	19

NUMBER OF EQUATIONS WITH k SIGNIFICANT FACTORS

(not including zero beta portfolio)

First 250 Trading Days

k	# of eq.
0	1
1	8
2	10
3	24
4	21
5	19
6	17

Second 250 Trading Days

k	# of eq.
0	2
1	11
2	10
3	22
4	21
5	15
6	19

Note: Significance level is .05 for two-tailed T-test.

TABLE 4- 20

SUMMARY INFORMATION

Test Period: 2c

Number of Trading Days: 500

Number of Securities: 100

UNIVARIATE TESTS OF APT

First 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	0
$\hat{b}_o = 0$	95
$\hat{b}_o = 1$	21

Second 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	0
$\hat{b}_o = 0$	86
$\hat{b}_o = 1$	15

NUMBER OF EQUATIONS WITH k SIGNIFICANT FACTORS

(not including zero beta portfolio)

First 250 Trading Days

k	# of eq.
0	1
1	7
2	7
3	25
4	21
5	23
6	15

Second 250 Trading Days

k	# of eq.
0	0
1	2
2	13
3	13
4	28
5	30
6	14

Note: Significance level is .05 for two-tailed T-test.

TABLE 4-21

SUMMARY INFORMATION

Test Period: 3a

Number of Trading Days: 500

Number of Securities: 100

UNIVARIATE TESTS OF APT

First 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	2
$\hat{b}_0 = 0$	85
$\hat{b}_0 = 1$	20

Second 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	0
$\hat{b}_0 = 0$	87
$\hat{b}_0 = 1$	15

NUMBER OF EQUATIONS WITH k SIGNIFICANT FACTORS

(not including zero beta portfolio)

First 250 Trading Days

k	# of eq.
0	1
1	6
2	22
3	19
4	20
5	14
6	18

Second 250 Trading Days

k	# of eq.
0	2
1	5
2	7
3	28
4	19
5	21
6	18

Note: Significance level is .05 for two-tailed T-test.

TABLE 4-22

SUMMARY INFORMATION

Test Period: 3b

Number of Trading Days: 500

Number of Securities: 100

UNIVARIATE TESTS OF APT

First 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	0
$\hat{b}_0 = 0$	79
$\hat{b}_0 = 1$	11

Second 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	0
$\hat{b}_0 = 0$	89
$\hat{b}_0 = 1$	17

NUMBER OF EQUATIONS WITH k SIGNIFICANT FACTORS

(not including zero beta portfolio)

First 250 Trading Days

k	# of eq.
0	2
1	11
2	17
3	14
4	22
5	25
6	9

Second 250 Trading Days

k	# of eq.
0	0
1	4
2	12
3	23
4	28
5	23
6	10

Note: Significance level is .05 for two-tailed T-test.

TABLE 4-23

SUMMARY INFORMATION

Test Period: 3c

Number of Trading Days: 500

Number of Securities: 100

UNIVARIATE TESTS OF APT

First 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	0
$\hat{b}_0 = 0$	83
$\hat{b}_0 = 1$	12

Second 250 Trading Days

Hypothesis	Rejections
$\hat{\alpha} = 0$	0
$\hat{b}_0 = 0$	87
$\hat{b}_0 = 1$	22

NUMBER OF EQUATIONS WITH k SIGNIFICANT FACTORS

(not including zero beta portfolio)

First 250 Trading Days

k	# of eq.
0	3
1	7
2	13
3	17
4	24
5	23
6	13

Second 250 Trading Days

k	# of eq.
0	0
1	5
2	10
3	20
4	23
5	23
6	19

Note: Significance level is .05 for two-tailed T-test.

different from one. Also reported are the number of equations which were found to have various numbers of significant coefficients on the arbitrage portfolios. The majority of securities have from two to five such coefficients.

The results are in substantial, though not complete, agreement with the predictions of the APT. Significant intercepts occur less than 1% of the time; the zero beta return is significant in about 90% of the trials, and differs from one in about 16% of the trials. Over 90% of the securities have significant coefficients on three or more of the arbitrage portfolios. There does not appear to be any significant deterioration in explanatory power over the second sub-periods. Moreover, the average R^2's agree with the initial communalities; for example, the communality estimated in sample 1a was 22.05% and the average R^2 from the first sub-period was 20.04%. These summary measures are to some extent misleading in that they understate the degree of conformity of the results with the APT. There is a definite tendency for the model to work quite well for the majority of the securities and work poorly for a minority. Typically, the intercept is insignificant at any conventional level, $\hat{b}_0$ is within one standard error of its predicted value, and several other coefficients are significant at any conventional level. The coefficient estimates are frequently in excess of three standard errors away from zero. The results in this section are generally in accord with the APT. That the model occasionally works poorly is not surprising; the theory is itself an approximation and is expected to have low explanatory power for some subset of the securities under consideration. While the predictions of the APT appear to be supported by the data, the results in this section do not account for cross-sectional dependencies in the estimates, and no general conclusion can be drawn about the central prediction of the theory, namely, an absence of arbitrage opportunities. This is the subject of the next section.

A Multivariate Test of the APT

In this section, consideration turns to the question of whether the intercepts and zero beta coefficients are jointly different from their predicted values. A good reference for the multivariate techniques employed in this section may be found in Timm (1975). The individual time-series estimates of eq. (3-30) can be brought together in matrix notation as

$$R = \Delta B + E, \tag{4-2}$$

where

$R = \langle \tilde{r}_{ti} \rangle$ = the 250 x 100 of security returns

$\Delta = \langle \tilde{\delta}_{tj} \rangle$ = the 250 x 8 matrix of augmented portfolio returns

$B = \langle \hat{b}_{ji} \rangle$ = the 8 x 100 matrix of estimated coefficients

$E = \langle \tilde{e}_{ti} \rangle$ = the 250 x 100 matrix of residuals.

It is assumed that

$$E[E] = \underline{0}$$

$$E[R] = \Delta B$$

$$V[R] = I_t \otimes \Sigma.$$

In this case

$$\hat{B} = (\Delta' \Delta)^{-1} \Delta' R, \tag{4-3}$$

and an unbiased estimate of Σ is

$$\hat{S} = \frac{(R - \Delta\hat{B})^1 (R - \Delta\hat{B})}{t-k-1}$$

$$= \frac{R'(I - \Delta(\Delta'\Delta)^{-1}\Delta')R}{t-k-1}. \tag{4-4}$$

The columns of $\hat{B}$ in (4-3) are the usual univariate OLS estimates. Even though the equations are distrubance related, the regressors are identical in each equation and thus OLS on each equation is efficient (Thiel (1971), p. 309). The hypothesis of interest is

$$H_0: \ \underline{\alpha}, \underline{b}_0 = \underline{0}, \underline{1} \qquad (4\text{-}5)$$

vs.

$$H_A: \ \underline{\alpha}, \underline{b}_0 \neq \underline{0}, 1$$

where $\underline{\alpha}$ is the first row of $\hat{B}$ and $\underline{b}_0$ is the second. If it is assumed that the rows of R possess multivariate normal distributions, then this hypothesis can be tested by comparing restricted and unrestricted sum of squares and cross-products. The hypothesis can be written as

$$H_0: \ CBM = \Gamma,$$

where C is the matrix of restrictions operating within the individual equations with M operating across equations. From Timm (1975), the error sums of squares and cross-products are

$$Q_e = M'R'[I - \Delta(\Delta'\Delta)^{-1}\Delta]\, RM,$$

and the sums due to the hypothesis are

$$Q_h = (C\hat{B}M - \Gamma)'\ [C(\Delta'\Delta)^{-1}C']^{-1}\ (C\hat{B}M - \Gamma).$$

The total sums of squares and cross-products under the null hypothesis is $Q_t = Q_e + Q_h$. With these definitions, four statistics based on the eigenvalues of $Q_e^{-1}Q_h$ or $Q_t^{-1}Q_h$ are available. Letting γ_i be the ordered eigenvalues of $Q_e^{-1}\ Q_h$, then the following statistics are estimated:

$$\text{Wilk's Lambda} = |Q_e||Q_t^{-1}| = \pi \frac{1}{1 + \gamma_i}$$

$$\text{Pillai's Trace} = \operatorname{tr}(Q_h Q_t^{-1}) = \Sigma \frac{\gamma_i}{1 + \gamma_i}$$

$$\text{Hotelling-Lawley Trace} = \operatorname{tr}(Q_e^{-1}Q_h) = \Sigma\, \gamma_i$$

$$\text{Roy's Greatest Root} = \gamma_1.$$

No general preference for any one of the four can be established; however, in the case where the null hypothesis is of the form of (4-5), the F approximation for Wilk's Lambda is exact and Roy's criterion is an upper bound. When the hypothesis is of the form H_0: $\underline{\alpha} = \underline{0}$ or H_0: $\underline{b}_0 = \underline{1}$, all four criteria are equivalent.

The results from the securities data for the three hypotheses $\underline{\alpha} = \underline{0}$, $\underline{b}_0 = \underline{1}$, and $\underline{\alpha}$, $\underline{b}_0 = \underline{0}$, $\underline{1}$ are reported in Tables 4-24 to 4-32. For first two hypotheses only Wilk's Lambda is reported because of the equivalence of the four criteria. Examining the results, the restriction $\underline{\alpha} = \underline{0}$ is not binding in any of the trials; thus it does not appear to be possible to form portfolios with no factor risk or net investment with significantly non-zero returns. On the other hand, the restriction $\underline{b}_0 = \underline{1}$ is binding in every case. This suggests the existence of portfolios with no factor risk and returns that exceed the zero-beta return. Such an operation requires a positive investment, however. This finding may be attributable to the difficulties in estimating the zero beta return described above. When the two hypotheses are combined, the results indicate some dependency between the estimates; however, no unambiguous conclusion concerning the validity of the hypothesis can be reached. Depending on the significance level and test statistic, the hypothesis appears to be binding in about one third of the trials, indicating significant though not complete agreement with the predictions of the APT.

Summary

The purpose of this chapter has been to elevate the APT from a relatively unstructured theory to a concrete model of security returns. Also developed were testable hypotheses of the APT with the emphasis placed on the conformity of the uniquely determined parts of the estimated structure with the predictions of the theory. The results of this positivist approach are strongly supportive of the basic content of the theory: it is not possible to form portfolios with no net investment and no factor

TABLE 4-24
SUMMARY INFORMATION

Test Period: 1a
Number of Trading Days: 500
Number of Securities: 100

FIRST 250 TRADING DAYS

TEST OF H_0: $\underline{\hat{\alpha}} = \underline{0}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lamba	.709	.663	93	150	.994

TEST OF H_0: $\underline{\hat{b}_0} = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.508	1.54	93	150	.007

TEST OF H_0: $\underline{\hat{\alpha}}, \underline{\hat{b}_0} = \underline{0}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.361	1.072	186	300	.296
Pillai's Trace	.781	1.041	186	302	.377
Hotelling-Lawley	1.376	1.102	186	298	.226
Roy's Greatest Root	1.575	1.575	93	151	.007

Note: F Statistic for Roy's Greatest Root is an upper bound.
F Statistic for Wilk's Lamba is exact.

TABLE 4-24 (continued)

SECOND 250 TRADING DAYS

TEST OF H_0: $\hat{\underline{\alpha}} = \underline{0}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.741	.563	93	150	.999

TEST OF H_0: $\hat{\underline{b}}_0 = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.380	2.636	93	150	< .001

TEST OF H_0: $\hat{\underline{\alpha}}, \hat{\underline{b}}_0 = \underline{0}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.281	1.429	186	300	.003
Pillai's Trace	.879	1.274	186	302	.031
Hotelling-Lawley	1.986	1.591	186	298	< .001
Roy's Greatest Root	1.638	2.660	93	151	< .001

TABLE 4-25
SUMMARY INFORMATION

Test Period: 1b
Number of Trading Days: 500
Number of Securities: 100

FIRST 250 TRADING DAYS

TEST OF H_0: $\hat{\underline{\alpha}} = \underline{0}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lamba	.679	.762	93	150	.923

TEST OF H_0: $\hat{\underline{b}}_0 = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.462	1.876	93	150	<.001

TEST OF H_0: $\hat{\underline{\alpha}}, \hat{\underline{b}}_0 = \underline{0}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.308	1.291	186	300	.025
Pillai's Trace	.863	1.233	186	302	.054
Hotelling-Lawley	1.686	1.350	186	298	<.011
Roy's Greatest Root	1.236	2.006	93	151	<.001

Note: F Statistic for Roy's Greatest Root is an upper bound.
F Statistic for Wilk's Lamba is exact.

TABLE 4-25 (continued)

SECOND 250 TRADING DAYS

TEST OF H_0: $\underline{\hat{\alpha}} = \underline{o}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.723	.617	93	150	.994

TEST OF H_0: $\underline{\hat{b}}_o = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.394	2.482	93	150	$<$ .001

TEST OF H_0: $\underline{\hat{\alpha}}$, $\underline{\hat{b}}_o = \underline{o}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.287	1.398	186	300	.005
Pillai's Trace	.874	1.260	186	302	.038
Hotelling-Lawley	1.925	1.542	186	298	$<$ .001
Roy's Greatest Root	1.568	2.546	93	151	$<$ .001

TABLE 4-26
SUMMARY INFORMATION

Test Period: 1c
Number of Trading Days: 500
Number of Securities: 100

FIRST 250 TRADING DAYS

TEST OF H_0: $\underline{\hat{\alpha}} = \underline{0}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lamba	.648	.875	93	150	.757

TEST OF H_0: $\underline{\hat{b}_0} = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.449	1.977	93	150	< .001

TEST OF H_0: $\underline{\hat{\alpha}}, \underline{\hat{b}_0} = \underline{0},\underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.287	1.399	186	300	.005
Pillai's Trace	.908	1.350	186	302	.010
Hotelling-Lawley	1.809	1,449	186	298	.002
Roy's Greatest Root	1.278	2.075	93	151	< .001

Note: F Statistic for Roy's Greatest Root is an upper bound.
F Statistic for Wilk's Lamba is exact.

TABLE 4-26 (continued)

SECOND 250 TRADING DAYS

TEST OF H_0: $\underline{\hat{a}} = \underline{0}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.649	.874	93	150	.758

TEST OF H_0: $\underline{\hat{b}}_0 = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.439	2.060	93	150	< .011

TEST OF H_0: $\underline{\hat{a}}, \underline{\hat{b}}_0 = \underline{0}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.286	1.420	186	300	.005
Pillai's Trace	.905	1.341	186	302	.012
Hotelling-Lawley	1.828	1.464	186	298	.002
Roy's Greatest Root	1.324	2.149	93	151	< .001

TABLE 4-27
SUMMARY INFORMATION

Test Period: 2a
Number of Trading Days: 500
Number of Securities: 100

FIRST 250 TRADING DAYS

TEST OF H_o: $\underline{\hat{\alpha}} = \underline{o}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lamba	.784	.444	93	150	>.999

TEST OF H_o: $\underline{\hat{b}}_o = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.341	3.122	93	150	<.001

TEST OF H_o: $\underline{\hat{\alpha}}$, $\underline{\hat{b}}_o = \underline{o}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.274	1.469	186	300	.002
Pillai's Trace	.855	1.213	186	302	.069
Hotelling-Lawley	2.179	1.745	186	298	<.001
Roy's Greatest Root	1.936	3.143	93	151	<.001

Note: F Statistic for Roy's Greatest Root is an upper bound.
F Statistic for Wilk's Lamba is exact.

TABLE 4-27 (continued)

SECOND 250 TRADING DAYS

TEST OF H_0: $\hat{\underline{\alpha}} = \underline{0}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.789	.431	93	150	>.999

TEST OF H_0: $\hat{\underline{b}}_0 = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.466	1.846	93	150	<.001

TEST OF H_0: $\hat{\underline{\alpha}}, \hat{\underline{b}}_0 = \underline{0},\underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.372	1.033	186	300	.399
Pillai's Trace	.737	.947	186	302	.657
Hotelling-Lawley	1.399	1.121	186	298	.190
Roy's Greatest Root	1.145	1.859	93	151	<.001

TABLE 4-28
SUMMARY INFORMATION

Test Period:	2b
Number of Trading Days:	500
Number of Securities:	100

FIRST 250 TRADING DAYS

TEST OF H_0: $\hat{\underline{\alpha}} = \underline{0}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lamba	.831	.328	93	150	>.999

TEST OF H_0: $\hat{\underline{b}}_0 = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.469	1.827	93	150	<.001

TEST OF H_0: $\hat{\underline{\alpha}}, \hat{\underline{b}}_0 = \underline{0}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.384	.988	186	300	.531
Pillai's Trace	.706	.886	186	302	.816
Hotelling-Lawley	1.366	1.094	186	298	.245
Roy's Greatest Root	1.163	1.889	93	151	<.001

Note: F Statistic for Roy's Greatest Root is an upper bound.
F Statistic for Wilk's Lamba is exact.

TABLE 4-28 (continued)

SECOND 250 TRADING DAYS

TEST OF H_0: $\underline{\hat{a}} = \underline{0}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.792	.425	93	150	>.999

TEST OF H_0: $\underline{\hat{b}}_0 = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.371	2.733	93	150	<.001

TEST OF H_0: $\underline{\hat{a}}, \underline{\hat{b}}_0 = \underline{0}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.298	1.342	186	300	.012
Pillai's Trace	.826	1.142	186	302	.154
Hotelling-Lawley	1.940	1.554	186	298	<.001
Roy's Greatest Root	1.695	2.752	93	151	<.001

TABLE 4-29
SUMMARY INFORMATION

Test Period: 2c
Number of Trading Days: 500
Number of Securities: 100

FIRST 250 TRADING DAYS

TEST OF H_0: $\underline{\hat{a}} = \underline{0}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lamba	.780	.455	93	150	>.999

TEST OF H_0: $\underline{\hat{b}}_0 = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.414	2.287	93	150	<.001

TEST OF H_0: $\underline{\hat{a}}, \underline{\hat{b}}_0 = \underline{0}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.325	1.215	186	300	.067
Pillai's Trace	.800	1.082	186	302	.270
Hotelling-Lawley	1.690	1.354	186	298	.010
Roy's Greatest Root	1.418	2.303	93	151	<.001

Note: F Statistic for Roy's Greatest Root is an upper bound.
F Statistic for Wilk's Lamba is exact.

TABLE 4-29 (continued)

SECOND 250 TRADING DAYS

TEST OF H_0: $\underline{\hat{\alpha}} = \underline{0}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.813	.371	93	150	≥.999

TEST OF H_0: $\underline{\hat{b}}_0 = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.436	2.083	93	150	<.001

TEST OF H_0: $\underline{\hat{\alpha}}, \underline{\hat{b}}_0 = \underline{0}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.356	1.092	186	300	.249
Pillai's Trace	.748	.969	186	302	.589
Hotelling-Lawley	1.522	1.219	186	298	.064
Roy's Greatest Root	1.298	2.108	93	151	.001

TABLE 4-30
SUMMARY INFORMATION

Test Period: 3a
Number of Trading Days: 500
Number of Securities: 100

FIRST 250 TRADING DAYS

TEST OF H_o: $\underline{\hat{\alpha}} = \underline{o}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lamba	.674	.780	93	150	.904

TEST OF H_o: $\underline{\hat{b}}_o = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.433	2.11	93	150	<.001

TEST OF H_o: $\underline{\hat{\alpha}}, \underline{\hat{b}}_o = \underline{o}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.293	1.369	186	300	.008
Pillai's Trace	.891	1.305	186	302	.020
Hotelling-Lawley	1.790	1.434	186	298	.003
Roy's Greatest Root	1.310	2.128	93	151	<.001

Note: F Statistic for Roy's Greatest Root is an upper bound.
F Statistic for Wilk's Lamba is exact.

TABLE 4-30 (continued)

SECOND 250 TRADING DAYS

TEST OF H_0: $\underline{\hat{a}} = \underline{0}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.819	.357	93	150	>.999

TEST OF H_0: $\underline{\hat{b}}_0 = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.468	1.835	93	150	<.001

TEST OF H_0: $\underline{\hat{a}}, \underline{\hat{b}}_0 = \underline{0},\underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.383	.992	186	300	.520
Pillai's Trace	.712	.897	186	302	.791
Hotelling-Lawley	1.360	1.090	186	298	.254
Roy's Greatest Root	1.143	1.856	93	151	<.001

TABLE 4-31
SUMMARY INFORMATION

Test Period: 3b
Number of Trading Days: 500
Number of Securities: 100

FIRST 250 TRADING DAYS

TEST OF H_0: $\underline{\hat{\alpha}} = \underline{0}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lamba	.753	.530	93	150	>.999

TEST OF H_0: $\underline{\hat{b}}_0 = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.491	1.675	93	150	.002

TEST OF H_0: $\underline{\hat{\alpha}}, \underline{\hat{b}}_0 = \underline{0}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.376	1.017	186	300	.446
Pillai's Trace	.740	.953	186	302	.638
Hotelling-Lawley	1.350	1.082	186	298	.273
Roy's Greatest Root	1.059	1.720	93	151	.002

Note: F Statistic for Roy's Greatest Root is an upper bound.
F Statistic for Wilk's Lamba is exact.

TABLE 4-31 (continued)

SECOND 250 TRADING DAYS

TEST OF H_0: $\underline{\hat{a}} = \underline{0}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.777	.462	93	150	>.999

TEST OF H_0: $\underline{\hat{b}}_0 = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.441	2.042	93	150	<.001

TEST OF H_0: $\underline{\hat{a}}, \underline{\hat{b}}_0 = \underline{0}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.341	1.148	186	300	.145
Pillai's Trace	.783	1.046	186	302	.363
Hotelling-Lawley	1.564	1.253	186	298	.042
Roy's Greatest Root	1.277	2.074	93	151	<.001

TABLE 4-32
SUMMARY INFORMATION

Test Period: 3c
Number of Trading Days: 500
Number of Securities: 100

FIRST 250 TRADING DAYS

TEST OF H_0: $\hat{\underline{\alpha}} = \underline{o}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lamba	.674	.780	93	150	.904

TEST OF H_0: $\hat{\underline{b}}_o = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.433	2.110	93	150	<.001

TEST OF H_0: $\hat{\underline{\alpha}}, \hat{\underline{b}}_o = \underline{o}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.293	1.369	186	300	.008
Pillai's Trace	.891	1.305	186	302	.020
Hotelling-Lawley	1.790	1.434	186	298	.003
Roy's Greatest Root	1.310	2.128	93	151	<.001

Note: F Statistic for Roy's Greatest Root is an upper bound.
F Statistic for Wilk's Lamba is exact.

TABLE 4-32 (continued)

SECOND 250 TRADING DAYS

TEST OF H_0: $\underline{\hat{\alpha}} = \underline{0}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.819	.357	93	150	>.999

TEST OF H_0: $\underline{\hat{b}}_0 = \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.468	1.835	93	150	<.001

TEST OF H_0: $\underline{\hat{\alpha}}, \underline{\hat{b}}_0 = \underline{0}, \underline{1}$

Test Statistic	Computed Value	Computed F Statistic	Numerator DF	Denominator DF	Significance Level
Wilk's Lambda	.383	.992	186	300	.520
Pillai's Trace	.721	.897	186	302	.791
Hotelling-Lawley	1.360	1.090	186	298	.254
Roy's Greatest Root	1.143	1.856	93	151	<.001

risk with significantly non-zero returns. Also established were the significance of the estimated multiple risk premia in ex-laining daily security returns. When the role of the zero beta portfolio in security returns was examined, the data were found to be in substantial agreement with the predictions of the theory, although significant departures were found to exist. Owing to the difficulties experienced in translated *ex post* occurrences into *ex ante* expectations, this lack of conformity may be attributable to the collinearity between the zero beta return and the risk premia--a phenomenon at odds with the assumptions of the theory.

Whether the incidences of non-conformity are economic or econometric in origin, the fundamental conclusion of this research remains unaltered. The Ross multi-factor Arbitrage Pricing Theory can be successfully translated into an empirical model of security returns, the estimated structure of which is relatively stable and in general accord with the predictions of the theory. On the other hand, the arbitrage model, like any model, is a vast over-simplification, and its value lies in the potential for explai-ing (or at least accounting for) observed events. In this sense, its utility in terms of research can only be evaluated with reference to other models. Because there are an infinite number of hypotheses equally compatible with any finite set of observations, it is not possible to objectively establish the superiority of one model out of all potential models. To make matters worse, with the exception of artificial experiments, the superiority of one model over any other particular model can be established only with the aid of *ad hoc* decision rules (e.g., Occam's Razor). In the next chapter, the arbitrage model is compared to one popular variant of the market model in a common application: event studies.

The purpose of the comparison is to both illustrate the use of the arbitrage model in research and to compare the performance of the competing models in market efficiency research.

CHAPTER V
AN EVENT STUDY COMPARISON OF THE MARKET MODEL AND THE ARBITRAGE MODEL

Introduction

Previous chapters have sufficed to establish the arbitrage model as a viable empirical alternative in security markets research. The purpose of this chapter is to compare the usefulness of the arbitrage model to that of the popular market model. This question can be addressed in part by noting that the arbitrage model is clearly the model of choice on theoretical grounds. Other than a vague (and misleading) similarity to the CAPM, the market model has little in the way of theoretical support. In fact, the validity of the arbitrage model is a necessary condition for the validity of the market model, while the validity of the CAPM is neither necessary nor sufficient. However, theoretical considerations notwithstanding, models are constructed to explain observed events and their usefulness can be evaluated only in terms of the class of phenomena under investigation. Further, the substitution of a more complex model for a simple one involves a cost/benefit tradeoff. Even though the APT is considerably less complex than the CAPM (in terms of the underlying premises), the market model is conceptually a good deal less complicated to employ.[1] The general phenomena which models of securities markets are designed to explain are the cross-sectional differences in security returns. This is a broad subject; hence, the examination of this chapter is restricted to a particular subset of this general class: the reaction of share prices to unanticipated information. The purpose of this chapter is to illustrate the use of the arbitrage model in financial research and to demonstrate that it is a viable alternative to current practice. Because the scope is limited to a fraction of the possible uses, no general conclusions concerning the superiority of one model or the other will be drawn.

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[1] From a practical standpoint, the difference between the arbitrage and market models is relatively minor, given current software availability. If a relatively small number (less than, say, 200) of securities are used to form the arbitrage portfolios, and a large number of equations are estimated, then, the machine cost difference may be minor as well.

The approach adopted in this chapter is to examine the behavior of the residual or unsystematic portion of security returns about an event date using a methodology pioneered by Fama et al. (1969).[2] This methodology and its variants have been very popular tools in market efficiency research and no attempt is made here to catalogue its applications. The empirical work of Jaffee (1974), Mandelker (1974), and Scholes (1972), along with the Brown and Warner (1980) simulation study, are excellent examples. This chapter is further restricted in that a single version of each model is employed; the arbitrage model developed in the previous chapters in unrestricted form and a market model of the form

$$\tilde{r}_{jt} = \hat{\alpha}_j + \hat{b}_j \tilde{r}_{mt} + \tilde{e}_{jt}, \qquad (5\text{-}1)$$

where

$\tilde{r}_{jt}$ = return on the j^{th} security at time t,

$\hat{\alpha}_j, \hat{b}_m$ = OLS regression estimates for security j,

$\tilde{r}_{mt}$ = return on a broad-based market index at time t,

$\tilde{e}_{jt}$ = residual for security j at time t.

This is by no means the only possible version of the market model. Other formats include a risk premium approach (e.g., Jensen (1968)), a zero-beta approach such as Jaffee's (1974), or a restricted version such as that in Breen and Lerner (1972). As indicated below, certain other arbitrary choices concerning estimation must be made as well. This particular form has been used recently by Asquith and Kim (1982), Brown and Warner (1980), and Kalay (1982).

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[2] Actually, it is the difference between the observed and predicted returns which is studied; thus, the residual includes measurement errors, changes in structural parameters, and changes in investor attitudes.

Data for the Study

The time period covered by this study is 1630 trading days, beginning February 21, 1968, and running through September 16, 1974. This period was chosen to begin roughly $5^{1}/2$ years prior to the OPEC oil embargo. For this period, returns were assembled from the daily file for 100 securities arbitrarily selected as every tenth listing with a complete series. Also collected were the returns for the electric utility industry (SIC code 4911), the petroleum refining and oil field services groups (SIC codes 13 and 29), and the financial services group (SIC codes 60, 61, and 67).[3] The CRSP value-weighted index is used as a proxy for the market. In a fashion similar to that of Chapter IV, the data are broken down into three more or less arbitrary periods. The first 1250 trading days (about five years) are used as a base or estimation period; the following 100 trading days (about five months) are used as a comparison period, and the remaining 280 days comprise the event or test period. The exact dates are as follows:

Base Period: 2/21/68 - 3/15/73
Comparison Period: 3/16/73 - 8/07/73
Test Period: 8/08/73 - 9/16/74.

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[3] For each of these groups, a security was included if it maintained the same industry, classification throughout the period and it had a complete return series.

The first step was to obtain a six factor maximum likelihood solution for the 100 securities. As in the previous chapter, the weights for the arbitrage portfolios and their returns were then calculated. It is interesting to compare the returns on the CRSP indices with the arbitrage portfolio returns. This is accomplished by simply regressing the returns on the equal-weighted and value-weighted portfolios on the arbitrage and zero-beta portfolio returns over the base period. As in the previous chapter, the APT would predict a zero intercept and coefficient of one on the zero beta portfolio. As the results in Table 5-1 indicate, there is a highly significant relationship between the market indices and the arbitrage portfolios. The equal-weighted index behaves in accordance with the predictions of the arbitrage theory; however, the value-weighted index has a coefficient on the zero beta return that is lower than predicted. The value-weighted index is heavily invested in utilities, so this result may stem from the correlation described in Chapter V. These results suggest that the equal-weighted index is a superior proxy in a factor model sense; however, if the CAPM is the underlying intuition for the market model, the use of the value-weighted index is indicated. In the next section, the event study methodology is described for the two models.

An Event Study Methodology

In an event study, the researcher is interested in the difference between the predicted return on a group of securities and the actual return in a particular period. A favorable event, unanticipated by the market, should show up as a significant positive difference and vice versa. To obtain the predicted returns, the returns on each security are regressed on both the factor portfolios for the arbitrage model and the CRSP index for the market model using the base period data. Thus, for each security

TABLE 5-1

DEPENDENT VARIABLE: CRSP EQUAL-WEIGHTED INDEX

Variable	Parameter Estimate	Standard Error	Computed T Statistic	Significance Level
Intercept	-.0001	.00007	-1.478	.1398
Zero Beta	1.0094	.01665	60.607	<.0001
Arbitrage #1	.0038	.000054	69.979	<.0001
Arbitrage #2	.0038	.000052	73.644	<.0001
Arbitrage #3	.0036	.000051	70.900	<.0001
Arbitrage #4	.0034	.000047	70.749	<.0001
Arbitrage #5	.0024	.000047	52.042	<.0001
Arbitrage #6	.0018	.000047	39.257	<.0001

R^2 = .9227
DF = 1249

DEPENDENT VARIABLE: CRSP VALUE-WEIGHTED INDEX

Variable	Parameter Estimate	Standard Error	Computed T Statistic	Significance Level
Intercept	-.000025	.00006	-.416	.6777
Zero Beta	.7883	.01479	53.294	<.0001
Arbitrage #1	.0037	.000047	77.220	<.0001
Arbitrage #2	.0028	.000046	61.178	<.0001
Arbitrage #3	.0029	.000045	62.979	<.0001
Arbitrage #4	.0026	.000042	60.772	<.0001
Arbitrage #5	.0015	.000042	34.797	<.0001
Arbitrage #6	.0018	.000041	44.356	<.0001

R^2 = .9104
DF = 1249

in the group of interest, two regressions are run as follows:

$$\text{Market model:}\quad \tilde{r}_{jt} = \hat{\alpha}_j + \hat{b}_j\tilde{r}_{mt} + \tilde{e}_{jt} \qquad t = 1, . . ., 1250 \qquad (5\text{-}1)$$

$$\text{Arbitrage model:}\quad \tilde{r}_{jt} = \hat{\alpha}_j + \hat{b}_{oj}\tilde{\delta}_o + . . . + \hat{b}_{6j}\tilde{\delta}_6 + \tilde{e}_{jt},$$

$$t = 1, . . ., 1250. \qquad (5\text{-}2)$$

For the remaining 380 trading days, the resulting coefficients are multiplied by the actual returns on the factor portfolios to obtain a set of arbitrage model predicted returns and by the CRSP index return to generate market model predicted values. The predicted values are subtracted from the actual returns and a collection of errors for each model is the result; i.e.,

$$\text{Market model:}\quad \hat{e}_{jt} = \tilde{r}_{jt} - \hat{\alpha}_j - \hat{b}_j\tilde{r}_{mt} \qquad t = 1, . . ., 380 \qquad (5\text{-}3)$$

$$\text{Arbitrage model:}\quad \hat{e}_{jt} = \tilde{r}_{jt} - \hat{\alpha}_j - \hat{b}_{oj}\tilde{\delta}_o - . . . - \hat{b}_{6j}\tilde{\delta}_6 \quad t=1, . . 380. \qquad (5\text{-}4)$$

Next, for each day, the residuals for a particular group of securities are averaged, resulting in a 1 X 380 vector with each entry representing the average residual at time t (AR_t),

$$AR_t = \sum_{j=1}^{n} \hat{e}_{jt} \qquad t = 1, . . ., 380.$$

The hypothesis of interest takes the form

$$H_o:\ AR_t = 0$$

vs.

$$H_A:\ AR_t \neq 0.$$

A test of this hypothesis requires an estimate of the standard error of the average residuals. For each model, the first 100 days (the comparison period) are used to calculate $\overline{AR}$, the average of the average residuals, and ${}^{\sigma}AR$, the standard error of the average residual as follows:

$$\overline{AR} = \frac{1}{100} \sum_{t=1}^{100} AR_t \qquad (5\text{-}2)$$

$$\sigma_{AR} = \left(\frac{1}{99}\left[\sum_{t=1}^{100} AR_t^2 - 100\,\overline{AR}^2\right]\right)^{1/2}. \qquad (5\text{-}3)$$

If the residuals for day t represent a draw from a normal distribution and the draws are independent between days, then, under the null hypothesis, the ratio

$$\frac{AR_t}{\sigma_{AR}}, \qquad (5\text{-}4)$$

is T distributed with 99 degrees of freedom. The estimate of the standard error is obtained from a period different from the test period to insure independence of the numerator and denominator of (5-4). In particular, abnormal activity in the test period would result in an inflated estimate of σ_{AR}; hence, it is desirable to obtain the estimate from a "normal" period. Naturally, the choice of the 100 trading days subsequent to the base period as a representative period is arbitrary. An alternative would be to use the standard errors of the residuals from the base period. Such estimates would probably favor the arbitrage model as it is the nature of factor analysis to minimize the average residual variance. As it is measured here, σ_{AR} represents the error in the conditional forecast returns; thus, a lack of stability in the parameter estimates for either model would show up as an increased standard error.

In addition to the average residual, the cumulative average residual is also computed. The cumulative average residual at time t (CAR_t) is simply the sume of the average residuals over the t days

$$CAR_t = \sum_{i=1}^{t} AR_i. \qquad (5\text{-}5)$$

The behavior of the accumulated residuals provides evidence relating to market efficiency; subsequent to an event the residuals should be trendless if the information is fully impounded in security prices.

The average and cumulative residuals are computed over the 280 day test period for each of the three industry groups. For the oil group, the event is the oil embargo of October, 1973. For the utilities group, the event is the Consolidated Edison (Con Ed) dividend omission in April, 1974. No particular event is specified *a priori* for the financial services group; instead, the residuals are examined to determine if significant values correspond to events reported in the financial press, a check against errors of the first type.

Impact of the Oil Embargo on the Petroleum Refining and Oil Field Services Groups

In response to United States support of Israel in the Mid-east War, the OPEC cartel dramatically cut production, raised prices, and attempted to embargo petroleum products shipped to the United States. This action could reasonably be expected to effect the oil and oil-related industries. The exact announcement date is difficult to establish. The *Wall Street Journal* reported that widespread, though unconfirmed, rumors of price increases were circulating on October 10, 1973. Public announcements of substantial price increases and production cuts were reported on October 17, 18, and 19. In Table 5-2, the average residuals and associated T statistics are reported for both models with October 17 as the event date (day 0). The oil embargo appears to have had a substantial positive impact on these companies beginning on the tenth (day -4) and extending through the nineteenth (day +2). The results from the two models are quite similar; in fact, the cumulative abnormal performance over the period -4 to +2 is 8.09% for the arbitrage model and 8.89% for the market model and the standard errors are 52.6 basis points and 55.9 basis points respectively.

The cumulative average residuals (CAR) provide evidence of "windfall" profits associated with the oil embargo. By December 31, the market model

TABLE 5-2
SUMMARY INFORMATION

INDUSTRY GROUP: OIL
SIC CODE(S): 13, 29
NUMBER OF COMPANIES: 60
EVENT DATE: 10/17/73

STANDARD ERROR OF COMPARISON PERIOD AVERAGE RESIDUALS
ARBITRAGE MODEL: .00526126
MARKET MODEL: .00558909

EVENT PERIOD AVERAGE RESIDUALS AND T STATISTICS

ARBITRAGE MODEL		DAY RELATIVE TO	MARKET MODEL	
T	AR	EVENT DATE	AR	T
- .0630	-.0003	-30	-.0005	- .0947
.9045	.0047	-29	.0075	1.3407
.3898	.0021	-28	.0090	1.6091
- .0766	-.0004	-27	.0088	1.5733
.7858	.0041	-26	.0075	1.3342
- .2258	-.0012	-25	-.0042	- .7457
.7859	.0041	-24	-.0017	- .2965
.0084	.0000	-23	-.0010	- .1811
-1.4318	-.0075	-22	-.0004	- .0681
- .8149	-.0043	-21	.0029	.5178
1.1445	.0060	-20	-.0024	- .4262
2.2055	.0116	-19	.0098	1.7468
- .0933	-.0005	-18	.0059	1.0563
.2448	.0013	-17	.0041	.7303
1.5616	.0082	-16	.0030	.5423
-1.1090	-.0058	-15	-.0013	- .2247
.4643	.0024	-14	.0039	.7017
.2953	.0016	-13	.0018	.3178
.9545	.0050	-12	.0017	2.0892
.7927	.0042	-11	.0108	1.9285
- .5026	-.0026	-10	.0091	1.6261
- .3141	-.0017	- 9	.0015	.2747
- .5535	-.0029	- 8	-.0041	- .7254
-1.1695	-.0062	- 7	-.0016	- .2785
- .8326	-.0044	- 6	-.0044	- .7927
- .5206	-.0027	- 5	.0037	.6729
3.4795	.0183	- 4	.0143	2.5573
1.4322	.0075	- 3	.0071	1.2751
.7405	.0039	- 2	.0096	1.7245
1.5341	.0081	- 1	.0031	.5492
2.4813	.0131	0	.0183	3.2785
2.7250	.0143	+ 1	.0216	3.8715
2.9996	.0157	+ 2	.0149	2.6643
- .3465	-.0018	+ 3	.0008	.1520
- .1500	-.0008	+ 4	-.0031	- .5544
- .4610	-.0024	+ 5	-.0052	- .9370
.8287	.0044	+ 6	-.0011	- .1997
.6252	.0033	+ 7	.0004	.0643
-1.0699	-.0056	+ 8	-.0064	-1.1536
-1.1905	-.0063	+ 9	.0053	.9474
1.0213	.0054	+10	.0071	1.2697

CAR totalled 28.98% versus 17.50% for the arbitrage model. Interestingly, by the end of the test period (September 16, 1974), the market model CAR is 38.10% while the arbitrage model indicates only 2.83%. When these figures are adjusted by subtracting the residuals from the first five days in January (for reasons discussed below), the market model result is 27.72% compared to -.1% for the arbitrage model.

Impact of the Con Ed Dividend Omission on the Electric Utility Group

On April 23, 1974, Consolidated Edison, the nation's largest electric utility, announced that it would not be paying its regular quarterly dividend despite earnings sufficient to cover the expense. This announcement apparently surprised market participants; Con Ed stock opened late and fell from 18 to 12 1/4 by closing. This, along with poor earnings reports by a number of other utilities, resulted in the Dow Jones Utility Index reaching a fifteen year low. The residuals for the fifty-nine electric utilities on the CRSP file are reported in Table 5-3. Con Ed is not included in the sample because of its erratic stock price behavior.

Once again the results from the two models are similar. On the event date, there is evidence of a substantial negative abnormal return of 2%, followed by another drop of about 1.3%. The second day subsequent was relatively quite; however, days 3 through 5 show significant negative performance. Over the period 0 to +5, the cumulative residuals are -5.29% and -6.15% for the arbitrage and market models respectively. The standard error for the arbitrage model is again slightly less than that of the market model.

The behavior of the average residuals over the entire test period is erratic, with substantial abnormal performance on a number of days. Thus, it appears that the use of a different period to estimate the standard

TABLE 5-3
SUMMARY INFORMATION

INDUSTRY GROUP: ELECTRIC UTILITY
SIC CODE: 4911
NUMBER OF COMPANIES: 59
EVENT DATE: 04/24/74

STANDARD ERROR OF COMPARISON PERIOD AVERAGE RESIDUALS
ARBITRAGE MODEL: .00276072
MARKET MODEL: .00317469

EVENT PERIOD AVERAGE RESIDUALS AND T STATISTICS

ARBITRAGE MODEL		DAY RELATIVE TO		MARKET MODEL
T	AR	EVENT DATE	AR	T
.1666	.0005	-30	-.0031	- .9831
- .9542	-.0026	-29	-.0029	- .9272
.4546	.0013	-28	-.0013	- .4217
.6656	.0018	-27	.0026	.8247
- .6011	-.0017	-26	-.0004	- .1310
- .7955	-.0022	-25	.0007	.2311
.8616	.0024	-24	.0010	.3103
-1.6875	-.0047	-23	-.0074	-2.3300
-1.8622	-.0051	-22	-.0031	- .9828
- .4800	-.0013	-21	-.0015	- .4663
.3093	.0009	-20	-.0034	-1.0736
-1.1966	-.0033	-19	-.0031	- .9754
- .3213	-.0009	-18	.0004	.1295
1.3009	.0036	-17	.0013	.4083
.7996	-.0022	-16	-.0002	- .0774
.0418	.0001	-15	-.0013	- .3999
- .5284	-.0015	-14	-.0032	-1.0212
.1138	.0003	-13	-.0014	- .4368
- .4625	-.0013	-12	.0002	.0601
-1.2437	-.0034	-11	-.0017	- .5318
.3126	-.0009	-10	-.0024	- .7450
- .1905	-.0005	- 9	-.0029	- .9211
- .8597	-.0024	- 8	.0007	.2131
- .8797	-.0024	- 7	-.0021	- .6754
-1.3659	-.0038	- 6	-.0047	-1.4817
.8733	-.0024	- 5	-.0039	-1.2231
.9098	.0025	- 4	-.0009	- .2848
-1.0654	-.0029	- 3	-.0041	-1.2795
.2681	.0007	- 2	.0016	.4893
- .7846	-.0022	- 1	-.0039	-1.2362
-7.4857	-.0207	0	-.0187	-5.9051
-4.5659	-.0126	+ 1	-.0140	-4.4183
.3534	.0010	+ 2	-.0030	- .9393
-2.0137	-.0056	+ 3	-.0069	-2.1681
-2.6267	-.0073	+ 4	-.0093	-2.9369
-2.7851	-.0077	+ 5	-.0096	-3.0380
- .7399	-.0020	+ 6	-.0001	- .0227
- .9426	-.0026	+ 7	-.0016	- .5175
2.2567	.0062	+ 8	.0045	1.4309
- .2007	-.0006	+ 9	-.0024	- .7529
.9655	.0027	+10	.0013	.4181

error of the residuals is in fact warranted. These results generally confirm Pettway's (1978) findings of considerable instability subsequent to the dividend announcement. A final similarity between the two models is the cumulative residual over the entire period, amounting to -20.54% for the arbitrage model and -19.07% for the market model.

The Financial Services Group in the Period 8/73 - 9/74

In the previous two sections, residuals were examined about known (or hypothesized) event dates. Another potential use of this methodology is a general search of periods of disturbance in an attempt to identify specific shocks. This approach is illustrated for the financial services group in this section.

For the purposes of this section, the financial services group is defined as all firms in the CRSP file with SIC codes 60, 61 and 67. These codes correspond to banks, non-bank credit agencies (e.g., Savings and Loan Associatons), and financial holding companies. Even with this relatively broad definition, only thirty-one companies are available with complete returns data. The sample includes six national banks, three savings and loan associations, fifteen holding companies, six loan companies, and one mortgage banking company.

Because of its unique role in the economy, the financial services sector is of particular interest to researchers, especially with regard to its regulated status. A primary goal in the regulation of financial institutions is the early detection and prevention of bank failures. The extent to which market discipline regulates bank behavior has been studied by Pettway (1976) and Pettway and Sinkey (1980). In this context, the behavior of the residuals is examined in hopes of identifying shocks to particular institutions and the industry in general, in effect attempting to utilize the information and expectations of market participants to identify potential problems within the industry. The purpose of this section is to examine the residuals from both models for indications of abnormal performance and to determine if the signals correspond to actual reported events.

In the period studied, there are four instances of erratic behavior in the residuals: early December 1973 and early January, August, and September in 1974. The residuals for the December, August, and September periods are reported in Table 5-4 for both models. The January results are discussed in the next section.

The time period covered in this study was one of considerable uncertainty for the financial services industry, characterized by severe inflation, large-scale disintermediation, and several bank failures. The activity in each of the three periods is strongly associated with general market fears concerning the liquidity of the financial sector and specific policy actions by the Federal Reserve. As in the previous two sections, the results from the two models are similar; however, for this group, the standard error of the residuals is considerably less when the arbitrage model is employed.

In the first period, the behavior of the residuals is associated with several events. In the early part of the month, a number of large institutions raised their prime lending rate to a then remarkable 10%, a move counter to Federal Reserve anti-inflation policy and the objectives of the Nixon Committee on Interest and Dividends (CID). On December 10, the Federal Reserve cut the reserve requirement on large certificates of deposit from 14% to 8% in an effort to reduce the cost of funds to large institutions and stave off further rate increases. The cut corresponds to abnormal positive performance on December 11. Subsequent to the

TABLE 5-4

INDUSTRY GROUP: FINANCIAL SERVICES
SIC CODE(S): 60, 61 67
NUMBER OF COMPANIES: 31

STANDARD ERROR OF COMPARISON PERIOD RESIDUALS
ARBITRAGE MODEL: .00587216
MARKET MODEL: .00784628

AVERAGE RESIDUALS AND T STATISTICS

FIRST PERIOD: 12/03/73 - 12/18/73

ARBITRAGE MODEL			MARKET MODEL	
T	AR	DATE	AR	T
- .6724	-.0039	12/03	.0009	.1112
1.5349	.0090	12/04	.0057	.7302
.2147	.0013	12/05	-.0064	- .8118
-2.5280	-.0148	12/06	-.0241	-3.0765
-2.0080	-.0118	12/07	-.0118	-1.5044
- .9153	-.0054	12/10	-.0080	-1.0254
2.2188	.0130	12/11	.0170	2.1621
- .1896	-.0011	12/12	.0053	.6705
-1.4294	-.0084	12/13	.0016	.2008
-1.9798	-.0116	12/14	-.0105	-1.3406
-1.3435	-.0079	12/17	-.0044	- .5633
2.1983	.0129	12/18	.0024	.2998

TABLE 5-4 (continued)

SECOND PERIOD: 08/01/74-08/16/74

ARBITRAGE MODEL			MARKET MODEL	
T	AR	DATE	AR	T
1.0942	.0064	08/01	.0118	1.4989
1.9071	.0112	08/02	.0099	1.2592
-2.0631	-.0121	08/05	-.0165	-2.1038
-2.0655	-.0121	08/06	-.0148	-1.8911
-2.5621	-.0150	08/07	-.0213	-2.7206
.7367	.0043	08/08	.0132	1.6844
- .6213	-.0036	08/09	.0049	.6254
-2.5034	-.0147	08/12	-.0058	- .7382
.0276	.0002	08/13	.0035	.4469
- .2754	-.0016	08/14	.0036	.4578
1.5035	.0088	08/15	.0083	1.0630
-2.1349	-.0125	08/16	.0001	.0093

THIRD PERIOD: 08/23/74-09/10/74

ARBITRAGE MODEL			MARKET MODEL	
T	AR	DATE	AR	T
- .7400	-.0043	08/23	-.0045	- .5779
-2.8336	-.0166	08/26	-.0133	-1.6966
-1.9021	-.0112	08/27	-.0094	-1.1948
3.0415	.0177	08/28	.0153	1.9447
-3.7665	-.0221	08/29	-.0236	-3.0083
1.4256	.0084	08/30	-.0024	- .3052
.4769	.0028	09/03	.0158	2.0358
1.6529	.0097	09/04	.0164	2.0915
5.6596	.0332	09/05	.0164	2.0396
1.1166	.0066	09/06	.0003	.0332
-3.4729	-.0204	09/09	-.0062	- .7858
- .5799	-.0034	09/10	-.0044	- .5669

announcement, however, both the Fed and the CID exercised a bit of moral suasion by sending telegrams to the nation's institutions urging a moratorium on further increases, possibly accounting for the subsequent negative performance.

The negative performance in the second period is associated with renewed predictions of liquidity trouble in the industry. On August 6, the Treasury announced a large T-bill auction with rates well in excess of the legal maximum for bank deposits, and a deposit run-off was the expected result.

Continued erratic behavior is evidenced in the third period with a substantial positive shock around September 5. This activity is again associated with Federal Reserve actions; on September 4, the Fed removed the reserve requirement entirely on large certificates of deposit.

Despite Federal Reserve actions, the evidence from the arbitrage model suggests a cumulative abnormal performance over the entire period of -16.67%. The market model result differs, however, with a CAR of 4.06%. When the residuals from the first five days in January are subtracted out, the results are CARs of -20.93% and -6.11% for the arbitrage and market models respectively.

In the previous three sections, the behavior of the average residuals from the arbitrage and market models has been examined about various events. Based on these results, no clear preference for either model can be established. In fact, both models appear to perform well and the conclusions reached would be similar in either case. In the next section, an unanticipated result is reported which concerns the behavior of the residuals in the month of January.

Some Results on the January Effect

In recent years, the literature on empirical anomalies associated with the market model has grown considerably. The June 1983 issue of the Journal of Financial Economics is devoted entirely to abnormal returns which appear regularly in the securities data, and the introduction by Schwert (1983) provides a good summary of the issues and findings in this area. Papers by Keim (1983) and Brown *et al*. (1983) provide new evidence concerning the "size" effect; in particular, Keim documents that about 50% of the effect occurs in the month of January, with 25% concentrated in the first five trading days.

The time period studied in this chapter covers January of 1974, and the effect found by Keim shows up quite clearly when the maket model is used to generate the residuals. As reported in Table 5-5, significant abnormal performance is present in each of the three groups on the first five trading days of the month; moreover, the effect is detected in every group, despite the fact that the utilities group includes no small firms. When the residuals are accumulated over the five day period, the magnitude of the effect becomes apparent. For the oils and financials, the *excess* returns are both slightly greater than 10 percent when the market model is used, and this despite the fact that both groups include some of the largest firms in the economy. When the residuals are accumulated over the entire month, the importance of the first days is readily seen. In the utility and financial groups, the abnormal returns for the month are heavily concentrated in the first several days. For the oil group, the performance subsequent to January 8 is significantly negative, corresponding to announcements of pending IRS and Department of Energy audits and potential "windfall" profit taxes.

TABLE 5-5

AVERAGE AND CUMULATIVE AVERAGE RESIDUALS

UTILITY GROUP

ARBITRAGE MODEL			MARKET MODEL	
T	AR	DATE	AR	T
5.0155	.0138	01/02	.0178	5.6041
.4205	.0012	01/03	.0121	3.8145
2.6559	.0073	01/04	.0129	4.0668
.5559	.0015	01/07	.0082	2.5979
-1.2407	-.0034	01/08	.0005	.1498

OIL GROUP

ARBITRAGE MODEL			MARKET MODEL	
T	AR	DATE	AR	T
1.2875	.0068	01/02	.0244	4.3626
.0006	.0000	01/03	.0189	3.3730
1.0650	.0056	01/04	.0296	5.2888
1.6631	.0088	01/07	.0241	4.3131
1.5430	.0081	01/08	.0068	1.2175

FINANCIAL GROUP

ARBITRAGE MODEL			MARKET MODEL	
T	AR	DATE	AR	T
.2119	.0012	01/02	.0107	1.3690
2.4863	.0146	01/03	.0303	3.8598
- .1768	-.0010	01/04	.0146	1.8298
2.0326	.0119	01/07	.0243	3.0968
2.7000	.0159	01/08	.0218	2.7630

TABLE 5-5 (continued)

JANUARY CUMULATIVE AVERAGE RESIDUALS

FIRST FIVE TRADING DAYS

	ARBITRAGE MODEL	MARKET MODEL
UTILITIES	.0204	.0515
OILS	.0293	.1038
FINANCIALS	.0426	.1017

ENTIRE MONTH (22 TRADING DAYS)

	ARBITRAGE MODEL	MARKET MODEL
UTILITIES	.0130	.0493
OILS	-.0646	.0185
FINANCIALS	.0654	.1527

The evidence of Table 5-5 is necessarily idiosyncratic as only a single period is examined; however, for this period at least, the arbitrage model clearly does a superior job of accounting for the January phenomenon. If there is a consistent effect of the type detected in this period, then it is unlikely that a single index model will be adequate. Persistent abnormal performance of this magnitude is grossly incompatible with notions of market efficiency and no satisfactory explanation for its presence has been forthcoming. Evidence presented here suggests that the most likely explanation is sheer mis-specification in the market model pricing relation. This possibility illustrates a natural advantage of factor models: as an empirical matter, any consistent occurence of significant magnitude should be captured by one or more of the factors.

Summary

The purpose of this chapter has been to compare the performance of the arbitrage and market models in a particular (and very popular) type of research. The results are by no means definitive; event studies themselves constitute a very broad class of potential subjects and there is little possibility of establishing the absolute superiority of one model with regard to the entire class. On the other hand, some tentative conclusions can be drawn. First, the arbitrage model certainly can be used in such studies; in the events examined here, it performs at least as well as the market model in all three cases. Second, there appears to be a serious specification problem with the market model associated with the first part of the calendar year. Studies making use of January data and the market model must be viewed with some suspicion. It should be the case, and evidence presented in this chapter tends to confirm, that the arbitrage model is superior in this regard. This superiority can reasonably be expected to extend to other as yet unidentified regularities in security returns.

CHAPTER VI
RETURN, RISK AND ARBITRAGE: CONCLUSIONS

Introduction

Empirical research in securities markets is characterized by a body of commonly held tenets which collectively form the basis for the theory of finance. The most fundamental notion in this constellation is that of market efficiency. Securities markets are the most well-organized markets in the economy in terms of the availability of substitutes, the production of information about the commodities and the speed with which relative prices adjust. Financial economics as a field is neo-classical in orientation and its major pursuit is the extension of positive microeconomics to include uncertainty in a functionally useful way. The major attraction of the arbitrage theory is that it predicts nothing more than the simple economic proposition that markets characterized by large numbers of rational, utility maximizing individuals will operate to preclude any costless gains. When this premise is adopted *a priori*, several basic no-arbitrage propositions follow immediately:

1. Any portion of the riskiness of an individual asset which can be freely eliminated will not be relevant in the formulation of market required return; i.e., there will be no risk premium for diversifiable risk.

2. Assets will be priced as substitutes in well-diversified portfolios; thus, the relevant risk premium for a particular security is a function of its marginal impact on the riskiness of such a portfolio.

3. Any asset or portfolio whose riskiness can be completely diversified away will earn a return equal to the pure time-value of foregone consumption.

4. Market operations involving no risk and no foregone consumption (i.e., no net wealth) will have no return.

The intuitive reasonableness of these ideas may be the prime motivation behind two decades of the Capital Asset Pricing Model, but these propositions are all results of the CAPM. Thus, while the CAPM is consistent with this conventional wisdom, it is certainly not necessary.

Owing to the specificity of its assumptions, the CAPM is a much more powerful theory than is the APT, establishing the relative prices of every risky asset in the economy. It is this very power, however, which creates difficulties in application; a valid test literally requires that the relative values of all risky assets be observed. It is in this sense that the APT is a minimalist theory. Empirical verification of its predictions is a step which logically precedes more restrictive, and hence more powerful, theories and models.

Testing the Arbitrage Theory

In testing any theory, the researcher faces a sequence of decisions. The first step is the establishment of testable hypotheses about the theory. This step entails identifying alternatives to the predictions of the theory that, if discovered, would lead to its rejection. The second step is the experiment design in which the model is specified, statistical techniques of measurement are identified, and the data are assembled. The third step is the experiment itself and the observation of the results. For the arbitrage theory in particular, each of these steps has been addressed in previous chapters and it is useful to summarize them here.

Initially, the arbitrage theory assumes a k + 1 factor linear return generating function of the form

$$\tilde{r}_i = E_i + \tilde{\delta}_o + b_{i1}\tilde{\delta}_1 + \ldots + b_{ik}\tilde{\delta}_k + \tilde{e}_i, \tag{6-1}$$

where the residuals $(\tilde{e}_i)$ are independent both inter-temporally and cross-sectionally as well as being independent of the factors ($\tilde{\delta}_i$). Thus, the deviation of the return from its expectation is a linear function of the random values of the k common factors, the zero beta return, and an independent, residual component. If the basic no-arbitrage propositions outlined above are assumed to hold, then as shown in Chapter III, there is an approximate ex ante pricing relation of the form

$$E_i = \lambda_0 + b_{i1}\lambda_1 + \ldots + b_{ik}\lambda_k, \qquad (6\text{-}2)$$

where λ_0 is the expected zero beta return and $\lambda_1 - \lambda_k$ are the risk premia for the k factors. Substituting (6-2) into (6-3) results in the arbitrage model

$$\tilde{r}_i = (\lambda_0 + \tilde{\delta}_0) + b_{i1}(\lambda_i + \tilde{\delta}_1) + \ldots + b_{ik}(\lambda_k + \tilde{\delta}_k) + \tilde{e}_i. \qquad (6\text{-}3)$$

To this point, nothing has been said about the nature or number of the factors other than that they systematically influence security returns. However, there are two specific predictions which can be examined. First, the zero beta term must be a constant effect in every security return. Second, once the combined effect of the zero beta return and the factors are accounted for, there must be no other consistent effect in returns. Ignoring for the moment the exact specification of the model, these predictions can be brought together in a regression context as

$$\tilde{r}_i = \hat{\alpha}_i + \hat{b}_{i0}(\lambda_0 + \tilde{\delta}_0) + \hat{b}_{i1}(\lambda_i + \tilde{\delta}_0) + \ldots + \hat{b}_{ik}(\lambda_k + \tilde{\delta}_k) + \tilde{e}_i. \qquad (6\text{-}4)$$

The intercept term should not be different from zero, and $\hat{b}_{i0}$ should be one for the securities individually and jointly. Persistent findings to the contrary would lead to a rejection of the theory.

Testing the no-arbitrage propositions requires measurements of the factors and the zero beta return. Also needed is an estimate of the nunber of relevant factors. Because the factors are unspecified in the theory, they cannot be measured directly; instead, their existence must be inferred using latent variable techniques such as factor analysis. In the approach outlined in Chapter III, factor analysis is used to obtain an estimate of the approximate dimension of the arbitrage model and an initial set of factor coefficients. Using these coefficients, a portfolio is formed with unit sensitivity to a particular factor and no sensitivity to the others. When the portfolios thus formed are constrained to have minimum residual variance and use no net wealth, their returns can be interpreted as minimum variance estimates of the factor risk premia. Also, a zero beta portfolio is formed to have no sensitivity to any factor and minimum residual variance. This portfolio has positive investment and its return can be interpreted as a minimum variance estimate of the zero beta return. These portfolios are used as explanatory variables in regressions of the form specified in (6-4), and standard univariate and multivariate tests are performed for the intercept and zero beta coefficient.

The data selected to test the APT are daily security returns on AMEX and NYSE listed securities, and the tests are performed on individual securities. Testing the theory under these conditions is quite stringent in that daily returns in general are notoriously noisy, and, for any individual security, can be heavily influenced by unsystematic events. Even moderate success in ordering the daily data should be counted in the theory's favor.

The first problem that arises in estimating the model is technological; the numerical algorithm involved in obtaining a maximum likelihood factor solution is rather involved and the computational cost increases exponentially with the number of securities. Sample sizes in this study have thus been limited to one hundred securities at a time. Approximations to the ML solution are available and only experience and further research will determine their usefulness. The chief advantage of the ML procedure is the potential for objectively determining the number of significant factors; however, the results in Chapter IV indicate that the noise content in the ex post data makes results obtained from such tests suspect and necessitates a subjective decision. Again, the number of substantially important dimensions will ultimately be determined by experience.

The sample size limitation is itself a mixed blessing. If the entire market can be profiled with a relatively small, arbitrarily selected subset of the securities, then the case for the arbitrage model is improved. On the other hand, if extra-market effects are concentrated in certain types of securities (as they appear to be), then two related problems can arise. First, random sampling may not select a sufficient number of a certain type of security, resulting in an inadequate definition of some factor. Secondly, if only a small number of a particular type are included, the formation of the arbitrage portfolios may require extreme long or short positions in these securities to adequately resolve the data. The arbitrage portfolios thus formed will not be well-diversified and the measured risk premia may contain a substantial noise element. The lack of statistically significant premia reported in Chapter IV may be partially attributable to a lack of diversification in the arbitrage portfolios. For example, the correlation which arises between certain of the arbitrage portfolios and the zero beta return appears to be empirically related to heavy concentrations of public utilities within those portfolios. If this effect is truly interest rate related, then it is likely to show up regardless of sample size though only further research will resolve the issue. Finally, the problem of inferring *ex ante* expectations from *ex post* data is always present, and poor results may arise from either non-representative samples or instability in the model, and it is difficult to distinguish the two. Whether the problems--potential and actual--described in ths section are insurmountable is an empirical question, and the next section summarizes the results.

Empirical Findings for the Arbitrage Model

The first set of results stems from the factor analysis procedure and is of interest from an empirical point of view. The number of factors beyond the first which are significant was found to vary with the sample size and the criterion used. The decision to use six factors agrees with one of the criteria for the largest sample tested and is also consistent with cluster analysis results obtained by other authors. If anything, six factors would appear to be more than enough. Two results which consistently appear relate to the total systematic risk in the data and the existence of a single dominant factor. The communality estimates obtained from the factor analysis suggest that 20 to 25 percent of the total variance in daily returns is systematic and that about 75 percent of this portion can be explained by a single factor. The potential gain from a multifactor model is thus (approximately) an additional 5 percent. This gain is especially significant if it is concentrated in a subset of securities or relates to seasonal occurrences (such as the January effect). Measurements of the multiple correlation coefficients between the factors and the securities indicate that the first factor can be measured quite well and subsequent factors less so. This measurement problem was found to diminish as the sample size was increased. In order to improve the measurability of the smaller factors, the original solution was transformed using a weighted Varimax rotation. When a general factor is present, this rotation has the effect of spreading the variance more or less evenly over the first several factors.

Once the factor solutions were obtained, the portfolios were formed and their returns computed. In general, the zero beta returns and the majority of the arbitrage returns are insignificantly different from zero. However, as discussed in Chapter III, this determination cannot be counted as evidence for or against the APT. The arbitrage portfolios are not uniquely determined and different transformations will result in different findings. Moreover, the issue is whether these portfolios systematically influence security returns, not their own significance *per se*. An unanticipated result from this analysis was a persistent tendency for at least one of the arbitrage portfolios to exhibit a high degree of correlation with the zero beta portfolio. This empirical regularity is primarily a nuisance in this study, but this consistent behavior lends support to the idea that systematic interest rate risk (or some similar risk) may play a significant role in security returns.

The basic procedure used to examine the no-arbitrage propositions can be summarized as follows:

1. Using five years of data (the base period) and one hundred arbitrarily selected securities, obtain a Varimax rotated six factor ML solution.

2. Calculate the portfolio weights.

3. Using the two years subsequent to the base period (the test period), calculate the arbitrage and zero beta portfolio returns.

4. For each of the one hundred securities, regress the daily returns on the arbitrage portfolio returns in the first year and again in the second year.

5. For each regression, test whether the intercept is different from zero and the coefficient on the zero beta portfolio is different from one.
6. Across the one hundred securities, test whether the coefficients are jointly different from zero and one.

These steps are repeated in each of three different, non-overlapping, base periods with three samples per base period. Thus, three hundred securities with two regressions each are examined for a total of six hundred per base period or eighteen hundred in all.

The tests on the individual equations indicate substantial, though not complete, agreement with the predictions of the arbitrage theory. On the one hand, significant intercepts occur less than 1% of the time; however, the coefficient for the zero beta return differs from one in about 16% of the trials and is not different from zero in about 10% of the trials. As to whether the factors systematically influence security returns, over 90% of the securities were found to have significant coefficients on three or more of the arbitrage portfolios. Thus while the rejection rates run ahead of that which would be expected by chance, the procedure used to obtain estimates is quite restrictive and has the greatest potential for rejecting the theory. For example, pooling the securities into portfolios would almost certainly improve the results as would the use of over-lapping base and test periods.

When the cross-sectional dependency among the equations is examined, the hypothesis of a jointly zero intercept cannot be rejected in any case. The hypothesis of a coefficient on the zero beta portfolio jointly equal to unity is rejected in every case. Thus it does not appear possible to form pure arbitrage portfolios with significantly non-zero returns though zero beta portfolios can be found with different returns (at least *ex post*). Because there may be some dependence within the equations between the two parameters, their joint significance was also examined. The rejection rate in this case depends on the criterion employed. The longest confidence intervals (Pillai's Trace) lead to no rejections (at the 1% level) while the shortest intervals (Roy's Criterion) result in rejections 100% of the time, and no one criterion is uniformly most powerful. These results indicate general support for the arbitrage theory particularly in light of the _ex post_-_ex ante_ problem, limited sample sizes, and measurement difficulties associated with the zero beta portfolio. If nothing else, these findings certainly suggest that there is sufficient empirical validity in the arbitrage model to warrant further research and development.

Implementing the Arbitrage Model

While the tests of theory indicate some success in ordering the daily data, a question remains as to whether such a model, based as it is on factor analysis, is likely to be useful in empirical research. This question was addressed in part by comparing the arbitrage model with the model which has dominated empirical research in finance--the market model. The specific comparison in Chapter V involves the behavior of the errors in the conditional forecast returns about a market shock, i.e., an event study. Three industry groups were selected as a basis for the comparison: (1) petroleum refining and oil field services, (2) electric utilities, and (3) financial services. For the oil group, the event examined was the oil embargo of October, 1973; and, for the utility group, the Con Ed dividend omission of April, 1974 was examined. No particular event was pre-specified for the financial services group; instead, the residuals were examined and significant values were compared to events reported in the financial press.

Two desirable characteristics for models used in an event study would be a small standard error in the conditional forecasts and an ability to sharply resolve the information effect in the residuals. For the events studied here, the arbitrage model is slightly superior on both counts. In all three cases, the standard errors are smaller when the arbitrage model is used; in one of the three cases (financials), the standard error is 40% smaller. Also, the behavior of the residuals about the event dates is slightly more focused when the arbitrage model is employed; there appears to be a greater tendency for the market model to spread the event over several days. Nonetheless, for the events studied within the industries, the use of either model would lead to similar conclusions. The results from the event study do serve to establish that the arbitrage model works at least as well as the market model in this type of research.

The most interesting comparison of the two models emerges from a phenomenon that was relatively undocumented at the time this research was initiated: the January effect. For all three groups, the market model points to dramatic positive abnormal performance in the first five trading days of January, 1974. For the oil and financial groups, this abnormal performance amounts to about 10% over the five day period, or roughly 25 times greater than the anticipated total return. To date, this effect has been associated with firm size, but the results for the utilities are no less significant statistically. The effect may be proportional to total return, not a function of firm size. Whatever the explanation, the existence of such returns is clearly incompatible with any sort of market efficiency. The arbitrage model generates significant residuals in the same period, but the five day cumulative abnormal returns are from 2 1/2 to 3 1/2 times smaller. In this regard, the arbitrage model is a superior representation of the return generating function.

Conclusion

In the area of security markets research, two distinct theoretical approaches have arisen in the last decade. The first is the mean-variance approach of which the CAPM is a general equilibrium specification. The other, newer approach is the arbitrage theory. Both are consistent with commonly held notions about financial markets. The mean-variance approach requires highly restrictive assumptions on either investor behavior (quadratic preferences) or return distributions (normality). Neither restriction is particularly appealing on intuitive grounds, and it is unlikely that a valid test of the predictions from the theory will be forthcoming. The arbitrage theory is less restrictive and, as this research has demonstrated, subject to empirical verification.

Empirical research in market efficiency has been dominated by rather naive models loosely (and inappropriately) based on the mean-variance theory. A great deal is known about the behavior of the market model, and it is now difficult to separate the issues of potential mis-specification and market efficiency. If for no other reason, the arbitrage model developed in Chapters III and IV is a useful benchmark. Finally, the market model itself is something of a dead end. Most researchers would be willing to accept that individual security returns are systematically related to overall market movements and further evidence seems unnecessary. The more important and interesting question concerns the nature of such movements and the components which characterize the market. The arbitrage theory is a logical starting point for research addressing this question, and the evidence of this study indicates that it is a promising approach as well.

REFERENCES

H. Akaike. "Information Theory and an Extension of the Maximum Likelihood Approach." In V. N. Petrov and F. Csaki (eds.), 2nd International Symposium on Information Theory. Budapest: Akilseoniai-Kindo, 1973.

H. Akaike. "A New Look at the Statistical Model Identification." IEE Transactions on Automatic Control (December 1974), 716-723.

R. D. Arnott. "Cluster Analysis and Stock Price Comovement." Financial Analyst's Journal (November-December 1980), 56-62.

K. J. Arrow. "The Role of Securities in the Optimal Allocation of Risk-bearing." Review of Economic Studies (October 1964), 91-96.

P. Asquith and E. H. Kim. "The Impact of Merger Bids on the Participating Firm's Security Holders." Journal of Finance (December 1982), 1209-1228.

R. Ball. "Anomalies in Relationships Between Securities' Yields and Yield Surrogates." Journal of Financial Economics (June/September 1978), 103-126.

R. W. Banz. "The Relationship Between Return and Market Value of Common Stock." Journal of Financial Economics (March 1981), 3-18.

S. Basu. "Investment Performance of Common Stocks in Relation to their Price-Earnings Ratios: A Test of the Efficient Market Hypothesis." Journal of Finance (June 1977), 663-682.

F. Black. "Capital Market Equilibrium with Restricted Borrowing." Journal of Business (July 1972), 444-455.

W. J. Breen and E. M. Lerner. "On the Use of β in Regulatory Proceedings." Bell Journal of Economics (Autumn 1972), 612-621.

M. J. Brennan. "Discussion." Journal of Finance (May 1981), 352-353.

E. F. Brigham. Fundamentals of Financial Management. New York: CBS College Press, 1983.

P. Brown, D.B. Keim, A.W. Kleidon and T. A. Marsh. "Stock Return Seasonalities and the Tax-Loss Selling Hypothesis." Journal of Financial Economics (December 1983), 105-127.

S. J. Brown and J. B. Warner. "Measuring Security Price Performance." Journal of Financial Economics (December 1980), 205-258.

R. B. Cattell. "The Scree Test for the Number of Factors." *Multivariate Behavior Research* (April 1966), 245-276.

Center for Research in Security Prices. *CRSP Stock Files User's Guide*. Chicago: Graduate School of Business, University of Chicago, 1983.

N. Chen and J. Ingersoll. "Exact Pricing in Linear Factor Models with Finitely Many Assets." *Journal of Finance* (June 1983), 985-988.

E. E. Cureton and S. A. Mulaik. "The Weighted Varimax Rotation and the Promax Rotation." *Psychometrika* (June 1975), 183-195.

G. Debreu. *Theory of Value*. New York: John Wiley and Sons, Inc., 1959.

E. J. Elton and M. J. Gruber. "Homogeneous Groups and the Testing of Economic Hypotheses." *Journal of Financial and Quantitative Analysis* (January 1970), 581-602.

E. J. Elton and M. J. Gruber, "Improved Forecasting through the Design of Homogeneous Groups." *Journal of Business* (October 1971), 432-450.

E. J. Elton and M. J. Gruber. "Estimating the Dependence Structure of Share Prices--Implications for Portfolio Selection." *Journal of Finance* (December 1973), 1203-1232.

E. F. Fama. "Efficient Capital Markets: A Review of Theory and Empirical Work." *Journal of Finance* (May 1970), 383-417.

E. F. Fama, L. Fisher, M. C. Jensen and R. Roll. "The Adjustment of Stock Prices to New Information." *International Economic Review* (February 1969), 1-21.

E. F. Fama and J. D. MacBeth. "Risk, Return, and Equilibrium: Empirical Tests." *Journal of Political Economy* (May 1973), 607-636.

D. E. Farrar. *The Investment Decision Under Uncertainty*. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1962.

J. L. Farrell. "Analyzing Covariation of Returns to Determine Homogeneous Stock Groupings." *Journal of Business* (April 1974), 186-207.

J. L. Farrell. "Homogeneous Stock Groups." *Financial Analyst's Journal* (May-June 1975), 50-62.

G. J. Feeney and D. D. Hester. "Stock Market Indicies: A Principal Components Analysis." In D. D. Hester and J. Tobin (eds.), *Risk Aversion and Portfolio Choice*. New York: John Wiley and Sons, 1967.

Federal Reserve Bulletin. Board of Governors of the Federal Reserve System. Washington: Division of Administrative Services, 1962-1980.

H. R. Fogler. "Common Sense on CAPM, APT, and Correlated Residuals." *Journal of Portfolio Management* (Summer 1982), 20-28.

H. R. Fogler, K. John, and J. Tipton. "Three Factors, Interest Rate Differentials and Stock Groups." Journal of Finance (May 1981), 323-335.

S. H. Friedberg, A. Insel, and L. E. Spence. Linear Algebra. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1979.

M. Friedman. "The Methodology of Positive Economics." In M. Friedman (ed.), Essays in Positive Economics. Chicago: University of Chicago Press, 1953.

I. Friend. "Discussion." Journal of Finance (May 1981), 350-352.

A Gehr. "Some Tests of the Arbitrage Pricing Theory." Journal of the Midwest Finance Association (1975), 91-105.

H. H. Harman. Modern Factor Analysis. Chicago: University of Chicago Press, 1976.

J. L. Horn and R. Engstrom. "Cattell's Scree Test in Relation to Bartlett's Chi-Square Test and Other Observations on the Number of Factors Problem." Multivariate Behavior Research (July 1979), 283-300.

W. G. Howe. Some Contributions to Factor Analysis. Report No. ORNL-1919. Oak Ridge: Oak Ridge National Laboratory, 1955.

G. Huberman. "A Simple Approach to Arbitrage Pricing Theory." Journal of Economic Theory (October 1982), 183-191.

J. E. Ingersoll. "Some Results in the Theory of Arbitrage Pricing." Unpublished manuscript (October 1982).

J. F. Jaffee. "Special Information and Insider Trading." Journal of Business (July 1974), 410-428.

M. C. Jensen. "The Performance of Mutual Funds in the Period 1945-1964." Journal of Finance (May 1968), 240-267.

J. D. Jobson. "A Multivariate Linear Regression Test for the Arbitrage Pricing Theory." Journal of Finance (September 1982), 1037-1042.

K. G. Jöreskog. "Some Contributions to Maximum Likelihood Factor Analysis." Psychometrika (December 1967), 443-482.

K. G. Jöreskog and A. S. Goldberger. "Factor Analysis by Generalized Least Squares." Psychometrika (September 1972), 243-260.

Journal of Financial Economics. North Holland Publishing Company. Amsterdam: North Holland, 1983.

G. G. Judge, W. E. Griffiths, R. C. Hill, and T. Lee. The Theory and Practice of Econometrics. New York: John Wiley and Sons, Inc., 1980.

H. F. Kaiser. "The Application of Electronic Computers to Factor Analysis." Education and Psychological Measurement (Winter 1960), 141-151.

A Kalay. "The Ex-Dividend Day Behavior of Stock Prices: A Re-Examination of the Clientele Effect." Journal of Finance (September 1982), 1059-1070.

K. J. Kalman and J. A. Pogue. "An Empirical Evaluation of Alternative Portfolio-Selection Models." Journal of Business (April 1967), 166-193.

D. B. Keim. "Size-Related Anomalies and Stock Return Seasonality." Journal of Financial Economics (December 1983), 13-32.

B. F. King. "Market and Industry Factors in Stock Price Behavior." Journal of Business (January 1966), 139-190.

T. C. Langetieg. "An Application of a Three-Factor Performance Index to Measure Stockholder Gains from Merger." Journal of Financial Economics (December 1978), 365-383.

D. N. Lawley. "The Estimation of Factor Loadings by the Method of Maximum Likelihood." Proceedings of the Royal Society of Edinburgh (December 1940), 64-82.

D. N. Lawley and A. E. Maxwell. Factor Analysis as a Statistical Method. New York: American Elsevier Publishing Company, Inc., 1971.

H. Levy. "Equilibrium in an Imperfect Market: A Constraint on the Number of Securities in the Portfolio." American Economic Review (September 1978), 643-657.

R. H. Litzenberger and K. Ramaswamy. "The Effect of Personal Taxes and Dividends on Capital Asset Prices." Journal of Financial Economics (June 1979), 163-195.

R. H. Litzenberger and K. Ramaswamy. "Dividends, Short Selling Restrictions, Tax-Induced Investor Clienteles and Market Equilibrium." Journal of Finance (May 1980), 469-485.

M. Livingston. "Industry Movements of Common Stocks." Journal of Finance (June 1977), 861-874.

W. P. Lloyd and R. A. Schick. "A Test of Stone's Two-Index Model of Returns." Journal of Financial and Quantitative Analysis (September 1977), 363-376.

G. Mandelker. "Risk and Return: The Case of Merging Firms." Journal of Financial Economics (December 1974), 303-335.

H. Markowitz. Portfolio Selection. New Haven: Yale University Press, 1959.

W. Menderhall and R. L. Schaeffer. Mathematical Statistics with Applications. Belmont, California: Wadsworth Publishing Company, Inc., 1973.

S. L. Meyers. "A Re-examination of Market and Industry Factors in Stock Price Behavior." Journal of Finance (June 1973), 695-705.

J. A. Ohlson and M. B. Garman. "A Dynamic Equilibrium for the Ross Aribtrage Model." Journal of Finance (June 1980), 675-684.

G. S. Oldfield and R. J. Rogalski. "Treasury Bill Factors and Common Stock Returns." Journal of Finance (May 1981), 337-350.

R. H. Pettway. "Market Tests of Capital Adequacy of Large Commercial Banks." Journal of Finance (June 1976), 865-875.

R. H. Pettway. "On the Use of β in Regulatory Proceedings: An Empirical Examination." Bell Journal of Economics (Spring 1978), 239-248.

R. H. Pettway and J. F. Sinkey. "Establishing On-Site Bank Examination Priorities: An Early Warning System Using Accounting and Market Information." Journal of Finance (March 1980), 137-150.

M. R. Reinganum. "The Arbitrage Pricing Theory: Some Empirical Results." Journal of Finance (May 1981a), 313-321.

M. R. Reinganum. "Misspecification of Capital Asset Pricing: Empirical Anomalies Based on Earnings Yields and Market Values." Journal of Financial Economics (March 1981b), 19-46.

R. Roll. "A Critique of the Asset Pricing Theory's Tests." Journal of Financial Economics (May 1977), 129-176.

R. Roll. "Ambiguity When Performance is Measured by the Securities Market Line." Journal of Finance (September 1978), 1051-1069.

R. Roll and S. A. Ross. "An Empirical Investigation of the Arbitrage Pricing Theory." Journal of Finance (December 1980), 1073-1101.

B. Rosenberg. "Extra-Market Components of Covariance in Security Returns." Journal of Financial and Quantitative Analysis (March 1974), 263-274.

B. Rosenberg and W. McKibben. "The Prediction of Systematic and Specific Risk in Common Stocks." Journal of Financial and Quantitative Analysis (March 1973), 317-333.

S. A. Ross. "The Aribtrage Theory of Capital Asset Pricing." Journal of Economic Theory (December 1976), 341-360.

S. A. Ross. "Return, Risk, and Arbitrage." In J. Friend and J. Bicksler (eds.), Risk and Return in Finance. Cambridge: Ballinger Co., 1977.

S. A. Ross. "The Current Status of the Capital Asset Pricing Model (CAPM)." Journal of Finance (June 1978), 885-899.

SAS Institute, Inc. SAS User's Guide: Statistics. Cary, N.C.: SAS Institute, Inc., 1982.

G. Schwarz. "Estimating the Dimension of a Model." Annals of Statistics (March 1978), 461-464.

M. Scholes. "The Market for Securities: Substitution versus Price Pressure and the Effects of Information on Share Prices." Journal of Business (April 1972), 179-211.

G. W. Schwert. "Size and Stock Returns, and Other Empirical Regularities." Journal of Financial Economics (December 1983), 3-12.

J. Shanken. "The Arbitrage Pricing Theory: Is It Testable." Journal of Finance (December 1982), 1129-1140.

W. F. Sharpe. "A Simplified Model of Portfolio Selection." Management Science (January 1963), 277-293.

W. F. Sharpe. "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk." Journal of Finance (September 1964), 425-442.

W. F. Sharpe. "Factors in NYSE Security Returns." Journal of Portfolio Management (Summer 1982), 5-19.

B. K. Stone. "Systematic Interest-Rate Risk in a Two-Index Model of Returns." Journal of Financial and Quantitative Analysis (November 1974), 709-725.

H. Theil. Principles of Econometrics. New York: John Wiley and Sons, Inc., 1971.

N. H. Timm. Multivariate Analysis. Belmont, Ca.: Wadsworth Publishing Company, Inc., 1975.

Wall Street Journal. Dow Jones and Company. New York: Dow Jones and Company, October, 1973-September, 1974.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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